Math 315 Homework 10

Due Friday, April 28

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following 2-player strategic game:

Player 2

Player 1		A	B	C
	A	$3, 1$	$2, 4$	$4, 3$
	B	$8, 3$	$4, 6$	$1, 1$
	C	$1, 5$	$5, 0$	$2, 2$

   (a) Formulate a linear program to find the mixed strategy maximin $\underline{v}_1$ for Player 1.

   (b) Use Excel to solve the linear program. What is $\underline{v}_1$ and what mixed strategy guarantees Player 1 at least an expected value of $\underline{v}_1$?

   (c) Formulate a linear program to find the mixed strategy maximin $\underline{v}_2$ for Player 2.

   (d) Use Excel to solve the linear program. What is $\underline{v}_2$ and what strategy guarantees Player 2 at least an expected value of $\underline{v}_2$?

2. In the following 3-player game, Player 1 chooses a row (A or B), Player 2 chooses a column (a or b), and Player 3 chooses a matrix (α or β).

	a	b
A	$1, 1, 0$	$2, 2, 3$
B	$0, 0, 1$	$3, 3, 0$

α

	a	b
A	$-1, -1, 2$	$2, 0, 2$
B	$0, 2, 2$	$1, 1, 2$

β

   (a) Are there any pure strategy Nash equilibria?

   (b) Find a mixed strategy Nash eqilibrium where Player 3 uses the pure strategy β and the other players use a mixed strategy.

   (c) Show that there is not a Nash equilibrium in which Player 1 uses the pure strategy B, Player 2 uses a mixed strategy in which a and b both have nonzero probability, and Player 3 uses a mixed strategy in which α and β have nonzero probability.

3. In the following 3-player game, Player 1 chooses a row (A or B), Player 2 chooses a column (a or b), and Player 3 chooses a matrix (α or β).

	a	b
A	$0,0,0$	$1,0,0$
B	$0,0,1$	$0,1,0$

α

	a	b
A	$0,1,0$	$0,0,1$
B	$1,0,0$	$0,0,0$

β

(a) Find all pure strategy Nash equilibria for this game.

(b) Find all mixed strategy Nash equilibria where Player 1 uses both A and B with nonzero probability, Player 2 uses both a and b with nonzero probability, and Player 3 uses both α and β with nonzero probability.

Hints:

i. There is more than one such Nash equilibrium; in fact there are infinitely many such Nash equilibria.

ii. To find all of the mixed strategy Nash equilibria of this type, assume that Player 1 uses the strategy $xA + (1-x)B$, Player 2 uses $ya + (1-y)b$, and Player 3 uses $z\alpha+(1-z)\beta$. At the Nash equilibria, Player 1 will be indifferent between A and B, Player 2 will be indifferent between a and b, and Player 3 will be indifferent between α and β.

4. For each of the following games, find all the Nash equilibria (pure and mixed) and find all evolutionarily stable strategies.

(a)

		Population: Dove	Population: Hawk
Mutation	Dove	$2,2$	$8,3$
	Hawk	$3,8$	$7,7$

(b)

		Population: Dove	Population: Hawk
Mutation	Dove	$2,2$	$0,1$
	Hawk	$1,0$	$7,7$