Math 315 Homework 2

Due Friday, February 17

Solutions must be written in LaTeX(except for problem 4(a)). You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. A two-player game is played as follows. Some quarters are placed on a table, arranged in an n-by-n grid. The two players alternate turns removing quarters. On each players turn, the player chooses one row or column of the grid, and then removes any number of quarters (at least one) from that row or column. The player to take the last quarter wins. If n is even, describe a winning strategy for Player 2.

2. Consider the standard game of Tic-Tac-Toe (played on a 3-by-3 board). Suppose that the game ends when the board is full, and at that point the winner is the player who first placed three X's or three O's in a row, column, or diagonal. (Normally, the game ends once a player has three X's or three O's in row, column, or diagonal, but here the game continues until the board is full to make it easier to count the number of strategies).

   (a) How many strategies does the first player have in this game? (Recall that a *strategy* for player i is a function that maps every vertex v in V_i to a chid of v.)

   (b) How many strategies does the second player have in this game?

3. Consider a game of 3-player Nim. The three players rotate turns (first Player 1, then Player 2, then Player 3, then back to Player 1, and so on). As in the normal Nim game, on a players turn, they choose a pile and can remove as many objects as they want from that pile (and they must remove at least one object). The player who takes the last object wins, and the other players lose. Since this game has three players, Von Neumann's Theorem (Theorem 3.13 from the textbook) does not apply, so it is possible that none of the players has a winning strategy.

   (a) Find a 3-player Nim game in which Player 1 has a winning strategy. (To specify the game of 3-player Nim, state the number of piles and how many objects are in each pile.) Describe Player 1's strategy.

   (b) Find 3-player Nim game in which Player 2 has a winning strategy. Describe Player 2's strategy.

   (c) Find a 3-player Nim game in which Player 3 has a winning strategy. Describe Player 3's strategy.

   (d) Find a 3-player Nim game in which none of the players has a winning strategy. Explain why none of the players has a winning strategy.

4. Consider the following game with 2 players. There are 7 pennies on a table. The players alternate turns, and on each player's turn, they can take 1 or 3 pennies from the table. If only 2 pennies are left, they can only take 1 penny. The player to take the last penny loses.

   (a) Draw the game tree for this game. (You do not need to do this part in LaTeX. You can draw the game tree on paper, and turn it in outside my office in the Learning Commons.)

   (b) Use the game tree to determine which player has a winning strategy. Describe that player's winning strategy.