Math 315 Homework 3

Due Friday, February 24

Solutions must be written in LaTeX(except for Problem 1). You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following game. First, Player 1 chooses between actions A and B. Then, with probability 1/3, Player 2 observes which action Player 1 has chosen and with probability 2/3 Player 2 does not observe the action Player 1 has chosen. In all cases (regardless of whether Player 2 saw Player 1's action or not), Player 2 chooses between actions a and b. If Player 1 chose A and Player 2 chose a, then they both get a payoff of 1. If Player 1 chose B and Player 2 chose b, then they also both get a payoff of 1. Otherwise, they both get 0. Draw the game tree for this game, and indicate the information sets on your game tree. (You do not need to do this part in LaTeX. You can draw the game tree on paper, and turn it in outside my office in the Learning Commons.)

2. Let $\succsim$ be a preference relation on the set of outcomes O that is complete, reflexive, and transitive.

   (a) Let $\approx$ be the binary relation on the set O defined as $x \approx y$ if and only if $x \succsim y$ and $y \succsim x$. Prove that $\approx$ is an equivalence relation (that is, prove that it is reflexive, symmetric, and transitive).

   (b) Let $\succ$ be the binary relation on the set O defined as $x \succ y$ if and only if $x \succsim y$ and $x \not\approx y$ (meaning it is not that case that $x \approx y$). Prove that $\succ$ is transitive and anti-symmetric. (A binary relation $\succ$ on the set O is *anti-symmetric* if for all $x, y \in O$, if $x \succ y$, then it is not the case that $y \succ x$.)

3. Prove Theorem 2.14 in the textbook: If a preference relation $\succsim$ satisfies the axioms of Continuity and Monotonicity and if $A \succsim B \succsim C$ and $A \succ C$, then there exists a unique number p with $0 \leq p \leq 1$ such that

   $$B \approx [p(A), (1-p)(C)]$$

   (*Hint:* The Continuity Axiom says p exists. You want to prove that p is unique. A standard method to prove uniqueness is to assume that there are two numbers p_1 and p_2 that both work, and then prove that $p_1 = p_2$.)

4. Suppose that a person whose preferences satisfy the von Neumann-Morgenstern axioms says that his preferences regarding outcomes A, B, C, and D satisfy

$$C \approx \left[\tfrac{3}{5}(A), \tfrac{2}{5}(D)\right], \qquad B \approx \left[\tfrac{3}{4}(A), \tfrac{1}{4}(C)\right], \qquad A \succ D$$

(a) Find a utility function representing this person's preferences. What is $u(A)$, $u(B)$, $u(C)$, and $u(D)$?

(b) Consider the following two lotteries:

$$L_1 = \left[\tfrac{2}{5}(A), \tfrac{1}{5}(B), \tfrac{1}{5}(C), \tfrac{1}{5}(D)\right] \quad \text{and} \quad L_2 = \left[\tfrac{2}{5}(B), \tfrac{3}{5}(C)\right]$$

Determine $u(L_1)$ and $u(L_2)$ using the utility function from (a). Which lottery (L_1 or L_2 would the person prefer?)

(c) Now suppose that the information we have about the player's preferences is the same, except that $D \succ A$ instead of $A \succ D$. In this case would the player prefer lottery L_1 or lottery L_2 (where L_1 and L_2 are as defined in part (b)). Explain your answer.