Math 315 Homework 6

Due Friday, March 31

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Find a two-player extensive form game (ie, as a game-tree) that has perfect information and no chance and that corresponds to the following strategic-form game:

		Player II			
		s_{II}^1	s_{II}^2	s_{II}^3	s_{II}^4
Player I	s_I^1	Draw	Draw	Draw	Draw
	s_I^2	Player I wins	Player I wins	Player II wins	Player II wins
	s_I^3	Player II wins	Draw	Player II wins	Draw

2. Consider the following game:

		Player 2			
		A	B	C	D
Player 1	A	$2,5$	$6,4$	$3,9$	$0,3$
	B	$8,4$	$4,0$	$6,3$	$1,3$
	C	$2,9$	$8,7$	$7,6$	$5,10$
	D	$5,8$	$7,5$	$4,9$	$0,7$

   Find all Nash equilibria for this game, including both pure and mixed strategies. (*Hint:* Remember that Nash equilibria only occur in rows and columns that are not strictly dominated.)

3. Consider the following game:

		Player 2		
		A	B	C
Player 1	A	$50,100$	$0,30$	$30,0$
	B	$80,0$	$50,20$	$20,10$
	C	$0,70$	$100,40$	$50,60$

   (a) Are there any pure strategy Nash Equilibria?

   (b) Is $\left(\frac{1}{2}B + \frac{1}{2}C,\ \frac{3}{11}A + \frac{8}{11}C\right)$ a Nash equilibrium? Explain your answer.

(c) Is $\left(\frac{1}{37}A + \frac{23}{37}B + \frac{13}{37}C, \frac{1}{6}B + \frac{5}{6}C\right)$ a Nash equilibrium? Explain your answer.

4. A two-player game is *symmetric* if the two players have the same strategy set $S_1 = S_2$ and the payoff functions satisfy $u_1(s_1, s_2) = u_2(s_2, s_1)$ for each $s_1, s_2 \in S_1$. Prove that the set of pure strategy Nash equilibria of a two-player symmetric game is a symmetric set: if (s_1, s_2) is a Nash equilibrium, then (s_2, s_1) is also a Nash equilibrium. (*Hint:* Use the definition of Nash equilibrium, Definition 4.17, from the textbook.)