

Math 315 Homework 7

Due Friday, April 7

Solutions must be written in L^AT_EX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Ann and Bob are playing the following 2-player game:

- First, they both choose “Heads” or “Tails” and reveal their choice simultaneously.
- Then, if both chose “Heads” or both chose “Tails”, Ann pays Bob \$20. If Bob chooses “Heads” and Ann chooses “Tails”, then Bob pays Ann \$30. If Bob chooses “Tails” and Ann chooses “Heads”, then Bob pays Ann \$10.

- (a) Assume that both players are risk-neutral (so their utilities can simply equal the monetary value). Write this game as a zero-sum game in strategic form.
- (b) What are \underline{v} and \bar{v} in pure strategies?
- (c) Find the value of the game in mixed strategies.
- (d) What is Ann’s optimal strategy?
- (e) What is Bob’s optimal strategy?
- (f) If both players use their optimal strategy, what is Ann’s expected payoff and what is Bob’s expected payoff?

2. Consider the following 2-player zero-sum game, in which a, b, c , and d are constants:

		Player 2	
		Strategy 1	Strategy 2
Player 1	Strategy 1	a	b
	Strategy 2	c	d

Suppose that $a > b$, $a > c$, $d > b$, and $d > c$ (this implies that the game does not have a value in pure strategies).

- (a) Determine Player 1’s optimal mixed strategy in terms of a, b, c , and d . (Recall that an optimal strategy in a zero-sum game is a strategy that corresponds to the value of the game).
- (b) Determine Player 2’s optimal mixed strategy in terms of a, b, c , and d .
- (c) What is the value of the game (in mixed strategies) in terms of a, b, c , and d ?

3. A 2-player zero-sum game is called a *fair game* if the value of the game (in mixed strategies) is 0. Consider the 2-player zero-sum game:

		Player 2	
		C	D
Player 1	A	-5	x
	B	4	-3

Find x so that the game is a fair game.

4. Consider the following zero-sum game:

		Player 2		
		D	E	F
Player 1	A	-2	1	2
	B	2	-1	0
	C	1	0	-2

- (a) What are \underline{v} and \bar{v} in pure strategies?
- (b) Formulate a linear programming problem to find \underline{v} in mixed strategies.
- (c) Formulate a linear programming problem to find \bar{v} in mixed strategies.
- (d) Use Excel to solve the linear program from part (b). What is \underline{v} ? What is Player 1's optimal strategy?
- (e) Use Excel to solve the linear program from part (c). What is \bar{v} ? What is Player 2's optimal strategy?
- (f) If both players use their optimal strategy, what is Player 1's expected payoff and what is Player 2's expected payoff?