Math 315 Homework 8

Due Friday, April 14

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following linear programming problem:

$$\text{Maximize } -30x_1 - 3x_2 - 11x_3 - 8x_4 - 6x_5 - 12x_6 \text{ subject to}$$

$$\begin{aligned} -2x_1 + x_2 - 2x_3 - x_4 - 2x_5 + x_6 &\leq 6 \\ -3x_1 + x_2 - x_3 - x_4 + x_5 - 2x_6 &\leq -5 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 &\geq 0 \end{aligned}$$

   (a) Determine the dual problem.

   (b) For the dual problem, graph the region corresponding to the inequalities. The minimum value will occur at one of the vertices of the region — what is the minimum value, and which vertex does it occur at?

   (c) What is the maximum value of the original linear programming problem?

2. Let G and $\hat{G}$ be two strategically equivalent two-player strategic-form games. Suppose that in both games Player 1 has two strategies $S_1 = \{A, B\}$ and Player 2 has two strategies $S_2 = \{C, D\}$. Suppose that the payoff functions in G are u_1 and u_2 and the payoff function in $\hat{G}$ are v_1 and v_2. Since G and $\hat{G}$ are strategically equivalent, there exists $\alpha_1, \alpha_2, \beta_1, \beta_2$ with $\alpha_1, \alpha_2 > 0$ such that $v_1 = \alpha_1 u_1 + \beta$ and $v_2 = \alpha_2 u_2 + \beta_2$. Call the mixed strategy payoff functions U_1, U_2, V_1, and V_2, respectively.

   (a) Suppose that $\sigma_1 = x_1 A + x_2 B$ is a mixed strategy for Player 1 and $\sigma_2 = y_1 C + y_2 D$ is a mixed strategy for Player 2. Prove the following:

$$\begin{aligned} V_1(\sigma_1, \sigma_2) &= \alpha_1 U_1(\sigma_1, \sigma_2) + \beta_1 \\ V_2(\sigma_1, \sigma_2) &= \alpha_2 U_2(\sigma_1, \sigma_2) + \beta_2 \end{aligned}$$

   (b) Prove Theorem 5.35 for 2-player 2×2 games: Prove that every mixed strategy Nash equilibrium (σ_1^*, σ_2^*) of G is also a mixed strategy Nash equilibrium of $\hat{G}$.

3. In a penalty kick in a game of soccer, one player (the kicker) attempts to kick the soccer ball into a net, while another player (the goalie) tries to stop the ball from going in the net. There is not enough time for the goalie to see where the ball is going before attempting to move towards it, so the goalie must guess which direction the ball will be going. For simplicity, we will assume that the kicker chooses to kick left or right (so we'll disregard kicking to the middle), and the goalie decides whether to block left or right. Also, for simplicity, we'll use "left" and "right" to mean the kicker's left and right, and we'll also assume that all players (both kickers and goalies) are right-handed. A study of professional soccer players observed the following:

   - If both kicker and goalie chose left, the kicker had a 58% chance of getting the ball in the net.
   - If both kicker and goalie chose right, the kicker had a 70% chance of getting the ball in the net.
   - If the kicker chose left and the goalie chose right, the kicker had a 93% chance of getting the ball in the net.
   - If the kicker chose right and the goalie chose left, the kicker had a 95% chance of getting the ball in the net.

   (a) Set this up as a strategic-form game. Use the probabilities as the payoffs.

   (b) Is this game strategically equivalent to a zero-sum game? Explain why or why not.

   (c) Find all Nash equilibria (both pure strategy and mixed strategy) for the game.

4. Consider the following 2-player strategic game:

Player 2

Player 1		C	D
	A	$4,3$	$1,0$
	B	$0,2$	$2,4$

   (a) Find the mixed strategy maximin $\underline{v}_1$ and minimax $\overline{v}_1$ for Player 1. What mixed strategy will guarantee Player 1 at least an expected value of $\underline{v}_1$?

   (b) Find the mixed strategy maximin $\underline{v}_2$ and minimax $\overline{v}_2$ for Player 2. What mixed strategy will guarantee Player 2 at least an expected value of $\underline{v}_2$?