Math 315: Practice Problems for Final Exam

1. Consider the following zero-sum game:

		Player 2	
		C	D
Player 1	A	5	-3
	B	-4	2

   (a) Find the mixed strategy maximin $\underline{v}$ and minimax $\overline{v}$.

   (b) What is Player 1's optimal strategy? What is Player 2's optimal strategy?

   (c) If both player's play their optimal strategy, what is Player 1's expected payoff? What is Player 2's expected payoff?

   (d) Find the Nash equilibrium for the game.

2. Consider the following 2-player strategic game:

		Player 2	
		C	D
Player 1	A	$4,3$	$1,5$
	B	$0,2$	$3,6$

   (a) Find the mixed strategy maximin $\underline{v}_1$ and minimax $\overline{v}_1$ for Player 1. What mixed strategy will guarantee Player 1 at least an expected value of $\underline{v}_1$?

   (b) Find the mixed strategy maximin $\underline{v}_2$ and minimax $\overline{v}_2$ for Player 2. What mixed strategy will guarantee Player 2 at least an expected value of $\underline{v}_2$?

3. Consider the following strategic game:

		Player 2		
		A	B	C
Player 1	A	$-3,11$	$5,4$	$-2,9$
	B	$0,5$	$4,-3$	$5,6$
	C	$2,1$	$-1,7$	$3,-1$

   (a) What is Player 1's pure strategy maximin $\underline{v}_1$? What is Player 1's pure strategy minimax $\overline{v}_1$?

   (b) What is Player 2's pure strategy maximin $\underline{v}_2$? What is Player 2's pure strategy minimax $\overline{v}_2$?

   (c) Formulate a linear program to find Player 1's mixed strategy maximin $\underline{v}_1$.

   (d) Formulate a linear program to find Player 2's mixed strategy maximin $\underline{v}_2$.

4. In the following 3-player game, Player 1 chooses a row (A or B), Player 2 chooses a column (a or b), and Player 3 chooses a matrix (α, β, or γ).

	a	b
A	$3,1,5$	$5,6,3$
B	$2,4,6$	$3,1,4$

α

	a	b
A	$1,2,3$	$2,4,3$
B	$3,5,2$	$5,2,3$

β

	a	b
A	$2,1,1$	$1,5,3$
B	$3,5,4$	$4,2,4$

γ

(a) What is Player 1's pure strategy maximin $\underline{v}_1$? What is Player 2's pure strategy maximin $\underline{v}_2$? What is Player 3's pure strategy maximin $\underline{v}_3$?

(b) What is Player 1's pure strategy minimax $\overline{v}_1$? What is Player 2's pure strategy minimax $\overline{v}_2$? What is Player 3's pure strategy minimax $\overline{v}_3$?

5. For each of the following games, determine whether the game is strategically equivalent to a zero-sum game. Explain your answer.

(a)

		Player 2	
		A	B
Player 1	A	$1,2$	$3,8$
	B	$-1,-4$	$4,10$

(b)

		Player 2		
		A	B	C
Player 1	A	$-3,11$	$1,3$	$-2,9$
	B	$0,5$	$4,-3$	$5,-5$
	C	$2,1$	$-1,7$	$3,-1$

6. Consider the following strategic game:

		Player 2		
		A	B	C
Player 1	A	$-1,10$	$8,0$	$4,-2$
	B	$0,-10$	$2,-4$	$3,8$
	C	$-2,2$	$6,4$	$5,-10$

(a) Is there a mixed strategy Nash equilibrium in which Player 1 uses strategies A and B with non-zero probability and does not use strategy C, and in which Player 2 uses strategies A and B with non-zero probability and does not use strategy C? Explain your answer.

(b) Find a mixed strategy Nash equilibrium in which Player 1 uses strategies A, B, and C with non-zero probability and in which Player 2 uses strategies A and C with non-zero probability and does not use strategy B. (There are infinitely many such strategies; you just need to find one.)

7. For each of the following games, find the Nash equilibria and the Evolutionarily Stable Strategies.

(a)

		Player 2	
		A	B
Player 1	A	$3,3$	$2,2$
	B	$2,2$	$1,1$

(b)

		Player 2	
		A	B
Player 1	A	$4,4$	$0,1$
	B	$1,0$	$2,2$

(c)

		Player 2	
		A	B
Player 1	A	$-1,-1$	$4,0$
	B	$0,4$	$2,2$

8. Consider the following game:

		Player 2	
		A	B
Player 1	A	$6,7$	$0,10$
	B	$10,0$	$3,5$

Suppose that the two players play a repeated version of this game in which after each round, they play again with probability λ (and with probability $1-\lambda$, they stop playing).

(a) Suppose that Player 1 and Player 2 both always choose A. What is Player 1's expected payoff and what is Player 2's expected payoff?

(b) Suppose that Player 1 chooses A for the first two rounds and afterwards always chooses B. Suppose that Player 2 always chooses B. What is Player 1's expected payoff and what is Player 2's expected payoff?

(c) Suppose that both players alternate between choosing A and B with Player 1 choosing A on the first round and Player 2 choosing B on the first round (on the second round Player 1 chooses B and Player 2 chooses A, and on the third round Player 1 chooses A and Player chooses B, and so on). What is Player 1's expected payoff and what is Player 2's expected payoff?

9. Consider the following game (this is essentially the Prisoner's Dilemma with different payoffs than we used in class):

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	$-1, -1$	$-3, 0$
	Defect	$0, -3$	$-2, -2$

Suppose that two players play this game infinitely many times with the payoff for each player being $\sum_{t=0}^{\infty} \lambda^t u_i^t$ with u_i^t being Player i's payoff for round t. Consider the Grim Strategy in which a player chooses Cooperate on each round until the other player chooses Defect. After the other player plays Defect, the Grim Strategy chooses Defect from then on. For what values of λ is (Grim strategy, Grim Strategy) a Nash equilibrium?

10. Consider the following coalition game:

Coalition	A	BC	AC	AB	ABC
Wealth	10	90	30	60	140

(a) Find the Shapley allocation for this coalition game.

(b) Is the allocation from part (a) a rational allocation. Explain your answer.

(c) Find the Nucleolus allocation for this coalition game.

11. A *constant-sum* game is a 2-player strategic game in which for all strategy choices of Player 1 and 2, the sum of their payoffs is equal to a constant C. Prove that a constant-sum game is strategically equivalent to a zero-sum game.