Math 315: Practice Problems for Midterm

1. Consider the following game:

   - There are two players.
   - The game begins with one pile of beans containing 5 beans.
   - On each player's turn, he or she removes either one or two beans from the pile of beans.
   - The player to take the last bean loses.

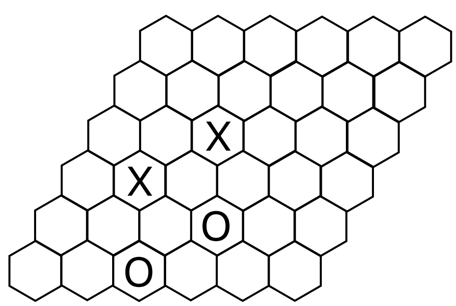
   Draw the game tree for this game. Use backward induction to determine which player has a winning strategy? Describe that player's winning strategy.

2. Consider a game of Nim with four piles of sizes 22, 19, 14, and 11.

   (a) Which player can be guaranteed to win this game (assuming the player uses correct strategy)? Explain your answer.

   (b) Suppose that the first player's move is to remove 6 beans from the heap of size 19. What should the second player's move be (in order to guarantee that the second player can win the game)?

3. Two players are playing 6×6 Hex. Player 1 is trying to create a path with X's from top to bottom, and Player 2 is trying to create a path with O's from left to right. They have reached the following position:



   It is now Player 1's turn. Determine where Player 1 should play next in order to guarantee that he can win the game. Explain why that move will guarantee that he can win.

4. Suppose that Sarah has a linear utility function u that represents her preferences regarding outcomes A, B, C, and D. Suppose that $u(A) = 0$, $u(B) = 30$, $u(C) = 80$, and $u(D) = 100$.

   (a) Would she choose outcome B or a lottery in which she has a 50% chance of outcome C and a 50% chance of outcome A?

   (b) Consider the lotteries $L_1 = [0.3(B), 0.7(D)]$ and $L_2 = [0.2(A), 0.3(B), 0.5(C)]$. Find $u(L_1)$ and $u(L_2)$. Does Sarah prefer lottery L_1 or lottery L_2?

   (c) Consider the compound lottery $\widehat{L} = [0.5(L_1), 0.5(L_2)]$. Find a simple lottery L_3 so that $\widehat{L} \approx L_3$. What is $u(\widehat{L})$?

   (d) Suppose that v is also a linear utility function representing Sarah's preferences and that $v(A) = 10$ and $v(D) = 60$. Find $v(B)$ and $v(C)$.

5. Suppose that u and v are two linear utility functions representing two players' preferences on outcomes $O = \{\$0, \$10, \$20, \$30\}$ with the values given below. Determine whether these players are risk averse, risk neutral, or risk seeking.

   (a) $u(\$0) = 0$, $u(\$10) = 40$, $u(\$20) = 80$, and $u(\$30) = 100$

   (b) $v(\$0) = 0$, $v(\$10) = 20$, $v(\$20) = 50$, and $v(\$30) = 100$

6. Consider the following game:

Player 2

Player 1		A	B	C	D	E
	A	$0,1$	$9,0$	$2,3$	$4,0$	$2,6$
	B	$7,9$	$7,3$	$1,7$	$1,7$	$4,5$
	C	$7,5$	$10,10$	$3,5$	$2,4$	$3,3$

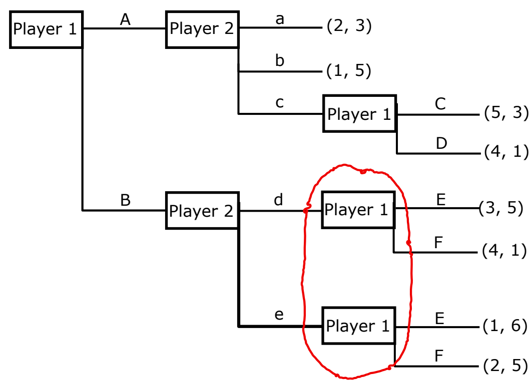
   (a) Eliminate any strictly dominating rows and columns. Repeat this process as long as either player has any strictly dominated strategies. Draw the resulting matrix after all strictly dominated strategies are removed.

   (b) What is Player 1's security level $\underline{v}_1$? Which strategy (or strategies) guarantee(s) Player 1 at least $\underline{v}_1$?

   (c) What is Player 2's security level $\underline{v}_2$? Which strategy (or strategies) guarantee(s) Player 2 at least $\underline{v}_2$?

   (d) What are the pure strategy Nash equilibria?

7. Consider the extensive-form game given by the following game tree. This game has one information set as indicated in the game tree.



Identify all of Player 1's strategies and all of Player 2's strategies. Write this game as a game in strategic-form.

8. Consider the following variant of Rock-Paper-Scissors:

		Rock	Paper	Scissors
Player 1	Rock	$0, 0$	$-1, 1$	$4, -4$
	Paper	$1, -1$	$0, 0$	$-4, 4$
	Scissors	$-4, 4$	$4, -4$	$0, 0$

(a) Is $\left(\frac{4}{9}\text{Rock} + \frac{4}{9}\text{Paper} + \frac{1}{9}\text{Scissors}, \frac{4}{9}\text{Rock} + \frac{4}{9}\text{Paper} + \frac{1}{9}\text{Scissors}\right)$ a Nash equilibrium for this game? Explain your answer.

(b) Is $\left(\frac{5}{9}\text{Rock} + \frac{4}{9}\text{Paper}, \frac{8}{9}\text{Rock} + \frac{1}{9}\text{Scissors}\right)$ a Nash equilibrium for this game? Explain your answer.

(c) Suppose that Player 1 uses the mixed strategy $\frac{5}{9}\text{Rock} + \frac{4}{9}\text{Paper}$ and that Player 2 uses the mixed strategy $\frac{8}{9}\text{Rock} + \frac{1}{9}\text{Scissors}$. What is Player 1's expected payoff? What is Player 2's expected payoff?

9. Consider the following strategic game:

Player 2

		C	D
Player 1	A	$1, 6$	$2, 8$
	B	$4, 5$	$1, 4$

Find all Nash equilibria for this game, including pure and mixed strategies.

10. Consider the following strategic game:

Player 2

Player 1		A	B
	A	$9,3$	$3,1$
	B	$8,2$	$6,8$
	C	$5,7$	$4,2$
	D	$4,5$	$7,9$

Find all Nash equilibria for this game, including pure and mixed strategies.

11. (a) Find a 2-player strategic-form game with 2 rows and 2 columns which has no pure strategy Nash equilibria.

    (b) Find a 2-player strategic-form game with 3 rows and 3 columns which has a pure strategy Nash equilibria, but in which iterative elimination of weakly dominated strategies yields a game with no pure strategy Nash equilibrium.

12. In the following 3-player game, Player 1 chooses a row (A or B), Player 2 chooses a column (a or b), and Player 3 chooses a matrix (α, β, or γ).

	a	b
A	$0,0,5$	$0,0,0$
B	$2,0,0$	$0,0,0$

α

	a	b
A	$1,2,3$	$0,0,0$
B	$0,0,0$	$1,2,3$

β

	a	b
A	$0,0,0$	$0,0,0$
B	$0,5,0$	$0,0,4$

γ

Find all pure strategy Nash equilibria for this game.

13. Suppose that a player indicates that he has the following preferences among lotteries with outcomes $O = \{A, B, C\}$:

$$A \succ B \succ C$$

$$[0.2(A), 0.7(B), 0.1(C)] \succ [0.3(A), 0.6(B), 0.1(C)]$$

Prove that there is not a linear utility function that represents this player's preferences.

14. Suppose that the functions u and v are both linear utility functions that represent a certain player's preferences over compound lotteries on a finite set of outcomes. Show that the function $u + v$ is also a linear utility function that represents the player's preferences.

15. Let $\succsim$ be a preference relation on the set of outcomes O that is complete, reflexive, and transitive. Let $\precsim$ be a relation on the set O defined as $x \precsim y$ if and only if $y \succsim x$. Prove that $\precsim$ is complete, reflexive, and transitive.