Math 316 Homework 1

Due Friday, February 12

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Show that if $n+1$ distinct integers are chosen from the set $\{1, 2, \ldots, 3n\}$, then there are always two which differ by at most 2.

2. (Exercise 24, Chapter 1) Find all 4-tuples (a, b, c, d) of distinct positive integers so that $a < b < c < d$ and
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$$
Hint: Look at the solution to Exercise 2 in Chapter 1.

3. Let S be a set of 17 points inside a cube of side length 1. Prove that there exists a sphere of radius 1/2 which encloses at least three of the points.

4. (Exercise 19, Chapter 2) Let $a_0 = 3$ and let $a_n = a_0 \cdot a_1 \cdot a_2 \cdot a_3 \cdots a_{n-1} + 2$ for $n \geq 1$. Prove that $a_n = 2^{2^n} + 1$.

5. Let $F_0, F_1, F_2, \ldots$ be the sequence defined by $F_0 = 0$ and $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. (Thus, F_n is the nth Fibonacci number.) By examining the terms of the Fibonacci sequence, make a conjecture about when F_n is divisible by 7 and then prove your conjecture.

Extra Credit:

Show that for any set consisting of 52 integers, there exist two integers a and b in the set such that $a^2 - b^2$ is divisible by 100.