Math 316 Homework 2

Due Friday, February 19

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (a) Find the number of integers between 40,000 and 80,000 in which no two adjacent digits are the same.

   (b) How many odd integers between 40,000 and 80,000 have no repeated digits?

2. In the card game bridge, a hand contains 13 cards. Determine the following probabilities. Give exact answers rather than decimal approximations (your answers can include binomial coefficients).

   (a) What is the probability that a bridge hand contains no Aces, Kings, Queens, Jacks, or Tens (that is, all cards are between 2 and 9)?

   (b) What is the probability that a bridge hand contains exactly 5 hearts?

   (c) What is the probability that a bridge hand contains 9 or more cards of the same suit?

3. (Exercise 38, Chapter 3) A host invites n couples to a party. She wants to ask a subset of the $2n$ guests to give a speech, but she does not want to ask *both* members of any couple to give speeches. In how many ways can she proceed?

4. Determine the number of ways to distribute 10 (indistinguishable) orange drinks, 1 lemon drink, and 1 lime drink to four thirsty students so that each student gets at least one drink, and the lemon and lime drinks go to different students.

5. Prove the following identity:

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

   Hint: Use a combinatorial proof. Let S be a set with three distinguished elements a, b, and c and count certain k-subsets of S.

Extra Credit:

Suppose that n of King Arthur's knights are seated at their customary round table. Three of them are chosen — all choices being equally likely — and are sent off to slay a troublesome dragon. Find the probability that at least two of the three had been sitting next to each other. Justify your answer.