Math 316 Homework 3

Due Friday, February 26

Solutions must be written in LaTeX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (Exercise 28, Chapter 4) Let n be a positive integer. Prove that $\binom{2n}{n}$ is even.

2. Let n be a positive integer. Prove the following identities:

   (a) $\displaystyle \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$

   (b) $\displaystyle n\binom{2n-1}{n-1} = \sum_{k=1}^{n} k\binom{n}{k}^2$

3. Let $n > 1$. Prove the following identities:

   (a) $\displaystyle \sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \frac{2^{n+1}-1}{n+1}$

   (b) $\displaystyle \sum_{a_1+a_2+a_3=n} \binom{n}{a_1, a_2, a_3}(-1)^{a_1+a_2} = (-1)^n$

4. Consider the following function:

$$f(x) = \frac{1}{\sqrt{1+3x}}$$

   Use Newton's Binomial Theorem to show that the power series for $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{(-1)^n \, 3^n}{2^{2n}} \binom{2n}{n} x^n$$

Extra Credit:

A company specializing in international trade has 70 employees. There are k different languages spoken by the employees. For any two employees A and B, there is a language that A speaks but B does not, and also a language that B speaks but A does not. What is the smallest number k such that this can occur? Explain how to assign the k languages to employees so that your answer works. (You do not need to prove that your answer is the smallest.)