

Math 316 Homework 6

Due Friday, April 1

Solutions must be written in \LaTeX . You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (Exercise 28, Chapter 5) Find a closed formula for $S(n, n - 3)$ for all $n \geq 3$.
2. Find a closed formula for $S(n, 4)$ for all $n \geq 4$. (*Hint*: Look at the solution to Exercise 1 in Chapter 5.)
3. Let k and n be positive integers with $k \leq n$. Prove that $p_k(n) \leq (n - k + 1)^{k-1}$.
4. Consider all permutations $p : [6] \rightarrow [6]$.
 - (a) Determine the number of such permutations for which p^2 is the identity permutation.
 - (b) Determine the number of such permutations for which p^3 is the identity permutation.
 - (c) Determine the number of such permutations for which p^6 is the identity permutation, but p^3 and p^2 are not the identity permutation.

Extra Credit

Let $n \in \mathbb{N}$ with $n \geq 2$. Let $F(n)$ be the number of all partitions of $[n]$ with no singleton blocks. Prove that $B(n) = F(n) + F(n + 1)$.