

Math 316 Homework 9

Due Friday, April 2

Solutions must be written in L^AT_EX. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (Exercise 24, Chapter 8) Let $a_0 = 1$ and $a_1 = 4$ and for $n \geq 2$ let a_n satisfy the following recurrence relation:

$$a_{n+1} = 8a_n - 16a_{n-1}$$

- (a) Find the generating function for the numbers a_n .
 - (b) Find a closed formula for a_n .
2. (Exercise 23, Chapter 8) Let $b_0 = 1$ and for $n \geq 1$ let b_n satisfy the following recurrence relation:

$$b_{n+1} = 3b_n + 2^n$$

- (a) Find the generating function for the numbers b_n .
 - (b) Find a closed formula for b_n .
3. (Exercise 25, Chapter 8) A certain kind of insect population multiplies so that at the end of each year, its size is the double of its size a year before, plus 1000 more insects. Assuming that originally we released 50 insects, how many of them will we have at the end of the n th year?
 4. Suppose we are creating fruit baskets containing apples, oranges, bananas, pears, and peaches with the following conditions:

- Each basket contains an even number of apples.
- The number of oranges in each basket is a multiple of three.
- Each basket contains at most 2 bananas, at most 1 pear, and at most 1 peach.

Let c_n be the number of such baskets that contain n pieces of fruit.

- (a) Find the generating function for c_n .
- (b) Use the generating function to find a formula for c_n .

Extra Credit:

Show that for any for any nonnegative integers m, r, n such that $m \leq r \leq n$,

$$\binom{n-m}{r-m} = \sum_{i=0}^m (-1)^i \binom{m}{i} \binom{n-i}{r}$$

(*Hint:* Use inclusion-exclusion and a combinatorial argument.)