Math 317: Answers to Practice Problems

Answers to problems from the textbook:
Many of the problems from the textbook have answers in the back of the book. Here are the answers to the problems that don't have answers in the back of the book.

6.3/2.17. (i) K_n is Eulerian if n is odd.

(ii) $K_{m,n}$ is Eulerian if m and n are both even.

(iii) The octahedron is the only Platonic solid that is Eulerian.

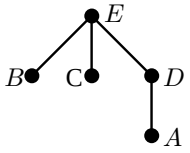
(iv) W_n is not Eulerian for any values of n.

(v) Q_k is Eulerian if k is even.

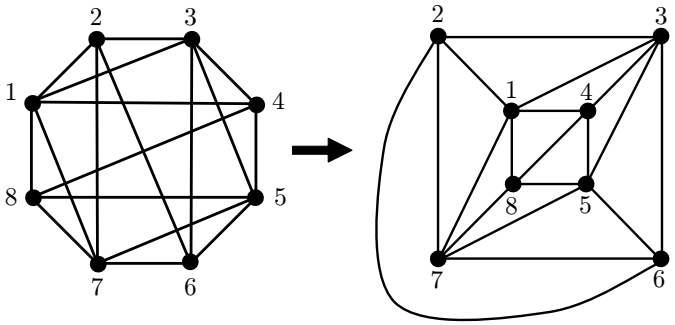
8.4/2.38. The shortest path from S to each vertex is indicated in the chart below:

Vertex	Distance	Shortest Path
A	1	$S \to A$
B	3	$S \to B$
C	5	$S \to B \to C$
D	3	$S \to A \to D$
E	4	$S \to A \to D \to E$
F	6	$S \to B \to F$
T	8	$S \to B \to F \to T$

11.2/3.21. The minimum spanning tree is:

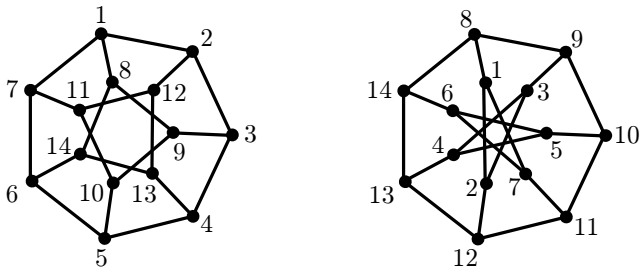


12.2/4.2. A planar drawing of the graph is shown below:

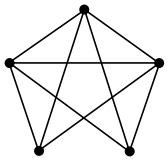


Answers to problems not from the textbook:

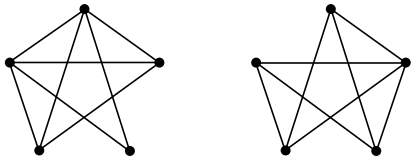
1. The two graphs are isomorphic, as shown in the following labeling:



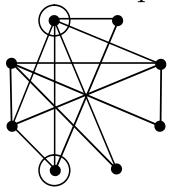
2. (a) There is one simple graph with 5 vertices and 9 edges. It is created by removing one edge from K_5:



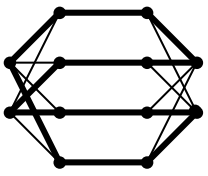
   (b) There are two simple graphs with 5 vertices and 8 edges. Both are created by removing two edges from K_5: one is created by removing two edges that share a vertex, the other is created by removing two disjoint edges.



3. The first graph is Eulerian, because every vertex has degree 4 (which is even). The second graph is not Eulerian, because it has a vertex of degree three and a vertex of degree five; these vertices are circled in the picture below:



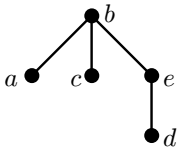
4. The first graph is not Hamiltonian. The second graph is Hamiltonian, and a Hamiltonian cycle is marked in bold below:



5. (a) K_n is Hamiltonian for all $n > 2$.

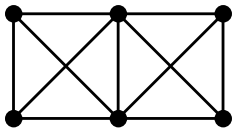
   (b) Complete bipartite graphs are Hamiltonian if and only if the two sets A and B both have the same number of vertices, and both have more than one vertex. Thus, $K_{n,n}$ with $n > 1$ is bipartite.

6. The minimum weight spanning tree for G is:



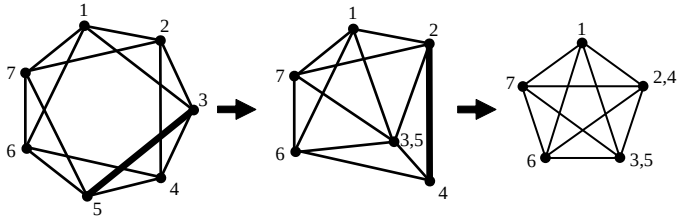
7. $\kappa(G) = 1$ and $\lambda(G) = 2$

8. There are many such graphs. One example is:

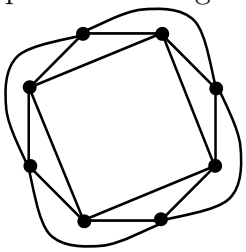


9. 30 edges, 12 faces

10. The first graph is not planar. As shown below, if we contract edge $\{3, 5\}$ and edge $\{2, 4\}$ (removing any double edges), we get K_5.



The second graph is planar. A planar drawing of the second graph is shown below:



The third graph is not planar. As shown below, if we remove the dotted edges, the resulting graph is a subdivided $K_{3,3}$. The circled vertices correspond to one set of vertices in $K_{3,3}$, and the squared vertices correspond to the other set. There is an edge or a subdivided edge between every circled vertex and every squared vertex.