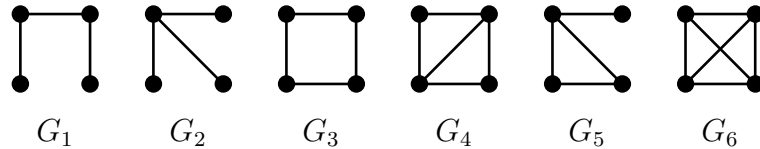# Math 317: Answers to Practice Problems

Answers to problems from the textbook:
Many of the problems from the textbook have answers in the back of the book. Here are the answers to the problems that don't have answers in the back of the book.

21.2/5.11. (i) The six graphs are:

$$G_1 \qquad G_2 \qquad G_3 \qquad G_4 \qquad G_5 \qquad G_6$$

The corresponding chromatic polynomials are:

$$
\begin{aligned}
P_{G_1}(k) &= k(k-1)^3 = k^4 - 3k^3 + 3k^2 - k \\
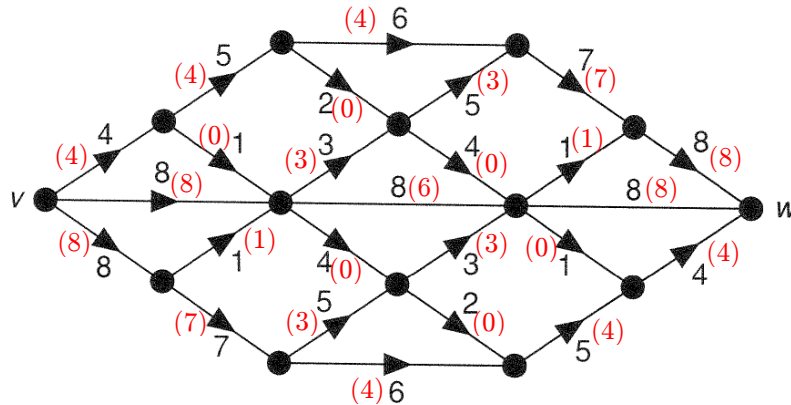P_{G_2}(k) &= k(k-1)^3 = k^4 - 3k^3 + 3k^2 - k \\
P_{G_3}(k) &= k(k-1)^2 + k(k-1)(k-2)^2 = k^4 - 4k^3 + 6k^2 - 3k \\
P_{G_4}(k) &= k(k-1)(k-2)^2 = k^4 - 5k^3 + 8k^2 - 4k \\
P_{G_5}(k) &= k(k-1)^2(k-2) = k^4 - 4k^3 + 5k^2 - 2k \\
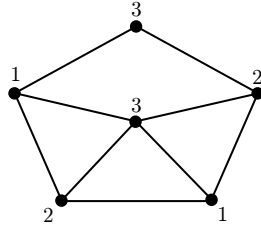P_{G_6}(k) &= k(k-1)(k-2)(k-3) = k^4 - 6k^3 + 11k^2 - 6k
\end{aligned}
$$

29.4/6.21. Here is a flow with value 20 (indicated in red):

This is a maximum flow, because there is a cut of capacity 20 (for example, all the edges incident to vertex v form a cut with capacity 20).

1

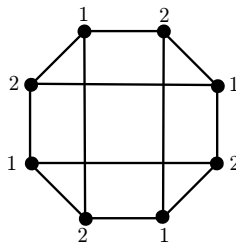**Answers to problems not from the textbook:**

1. The first graph G_1 has chromatic number $\chi(G_1) = 3$. Here is a coloring using the minimum number of colors:
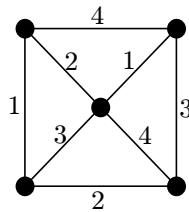

The second graph G_2 has chromatic number $\chi(G_2) = 4$. Here is a coloring using the minimum number of colors:
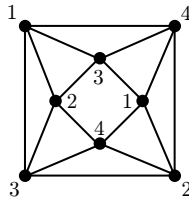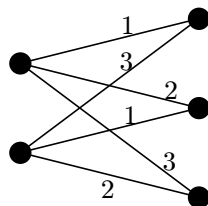

The third graph G_3 has chromatic number $\chi(G_3) = 2$. Here is a coloring using the minimum number of colors:
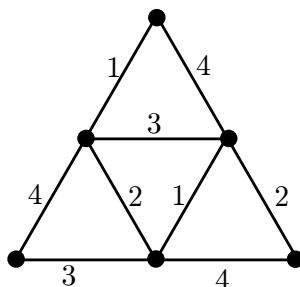

2. The first graph G_1 has chromatic index $\chi'(G_1) = 4$. Here is an edge-coloring using the minimum number of colors:

The second graph G_2 has chromatic index $\chi'(G_2) = 3$. Here is an edge-coloring using the minimum number of colors:

The third graph G_3 has chromatic index $\chi'(G_3) = 4$. Here is an edge-coloring using the minimum number of colors:

3. For the first graph G_1, the chromatic polynomial is:

$$P_{G_1}(k) = k(k-1)(k-2)^4$$

For the second graph G_2, the chromatic polynomial is:

$$P_{G_2}(k) = k(k-1)^2(k-2)^3 - k(k-1)(k-2)^3 + k(k-1)(k-2)(k-3)$$

For the third graph, G_3, the chromatic polynomial is:

$$P_{G_3}(k) = k(k-1)^4(k-2) - 3k(k-1)^2(k-2)^2 + 2k(k-1)(k-2)(k-3)$$

4. (a) States a, c, d,e, g, and h are persistent. States b and f are transient.

(b) States a and e are periodic with period 2. States c, d, g, and h are periodic with period 4. States b and f are aperiodic.

(c) No, it is not ergodic.

5. (a) All of the states are persistent.

(b) All of the states are aperiodic.

(c) Yes, it is ergodic.
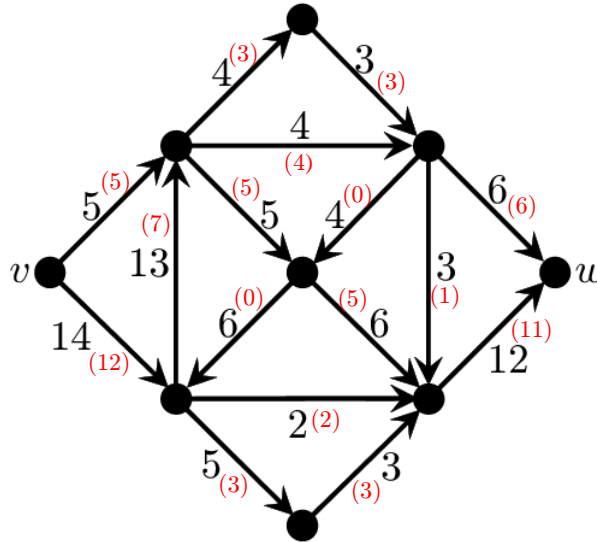
6. (a) The Markov Chain is:
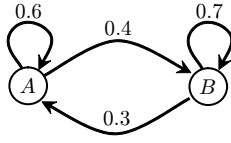
The transition matrix is:

$$T = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$$

(b) 42%

(c) 3/7 (Note: To find this, solve the equations $T\mathbf{x} = \mathbf{x}$ and $x_1 + x_2 = 1$.)

7. (a) A maximum flow is shown below in red. The value of the flow is value 17.

(b) A minimum cut for the network is indicated below — the capacities of the corresponding edges are circled. The capacity of the cut is 17.

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