Math 317 Homework 10

Due Friday, December 5

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Prove that $k^4 - 4k^3 + 3k^2$ is not a chromatic polynomial.

2. There are six graphs with chromatic polynomial $k^6 - 5k^5 + 10k^4 - 10k^3 + 5k^2 - k$. Find all six graphs.

3. Consider the following linear programming problem:

$$\text{Maximize } -30x_1 - 3x_2 - 11x_3 - 8x_4 - 6x_5 - 12x_6 \text{ subject to}$$

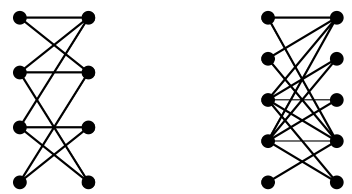
$$\begin{aligned} -2x_1 + x_2 - 2x_3 - x_4 - 2x_5 + x_6 &\leq 6 \\ -3x_1 + x_2 - x_3 - x_4 + x_5 - 2x_6 &\leq -5 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 &\geq 0,\ x_6 \geq 0 \end{aligned}$$

   (a) Determine the dual problem.

   (b) Solve the dual problem. (Graph the region corresponding to the inequalities; the minimum value will occur at one of the vertices of the region.)

   (c) What is the maximum possible value of $-30x_1 - 3x_2 - 11x_3 - 8x_4 - 6x_5 - 12x_6$ subject to the above inequalties?

4. For each of the following bipartite graphs determine whether the graph has a complete matching. Justify your answers.



5. Let T be a tree. Prove that T has a complete matching if and only if for every vertex v, exactly one component of $T - v$ has an odd number of vertices. (*Hint:* For the backwards direction, use induction on the number of vertices of T. Find a leaf with a neighbor of degree 2 and delete them both.)