Math 317 Homework 3
Due Friday, September 19

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Let G be a weighted graph with 7 vertices a, b, c, d, e, f, g and one edge between every pair of vertices. For each edge, the weight is given by the following table:

	a	b	c	d	e	f	g
a	0	20	27	51	40	25	46
b	20	0	22	44	30	34	52
c	27	22	0	42	15	10	53
d	51	44	42	0	50	37	57
e	40	30	15	50	0	17	55
f	25	34	10	37	17	0	54
g	46	52	53	57	55	54	0

   Find a minimum weight spanning tree for G.

2. (Exercise 5.10/2.44)

   (a) Let G be a simple graph on $2k$ vertices containing no triangles. Show, by induction on k, that G has at most k^2 edges. (This result is often called *Turán's extremal theorem*.)

   (b) Give examples of graphs with 4 vertices, 6 vertices, and 8 vertices for which this upper bound is achieved.

3. Prove that a knight cannot visit all the squares of a 7×7 chessboard exactly once using knight's moves and ending in the same square it started.

4. (Exercise 7.9/2.53) Let G be a Hamiltonian graph and let S be any set of k vertices in G. Prove that the graph $G - S$ has at most k components. (*Hint:* First prove that this holds if G is a cycle.)

5. Four people must cross a canyon at night on a fragile bridge. At most two people can be on the bridge at once. Crossing requires carrying a flashlight, and there is only one flashlight (which can cross only by being carried). Alone, the four people cross in 10, 5, 2, 1 minutes, respectively. When two cross together, they move at the speed of the slower person. In 18 minutes, a flash flood coming down the canyon will wash away the bridge. Can the four people get across in time? Prove your answer.