

Math 317 Homework 5

Due Friday, October 3

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (Exercise 13.4/4.16) Let G be the graph of a polyhedron (or polyhedral graph) in which every face is a pentagon or hexagon.
 - (a) Use Euler's formula to show that G must have at least 12 pentagonal faces.
 - (b) Prove, in addition, that if there are exactly three faces meeting at each vertex then G has exactly 12 pentagonal faces.
2. An **automorphism** ϕ of a simple graph G is a one-to-one mapping of the vertex set of G onto itself with the property that $\phi(v)$ and $\phi(w)$ are adjacent whenever v and w are adjacent (in other words, an automorphism is an isomorphism from G to G).
 - (a) How many automorphisms does K_4 have?
 - (b) How many automorphisms does $K_{3,3}$ have?
 - (c) Construct a simple graph with six vertices whose only automorphism is the identity.
 - (d) Construct a simple graph that has exactly three automorphisms. (*Hint*: Think of a rotating triangle with appendages to prevent flips.)
3. Play the planarity game at www.planarity.net. Submit a screenshot of a completed level 4 game. (Note: It is possible to skip directly to level 4, but if you have not played the game before, you may want to start with level 1.)