Math 317 Homework 7

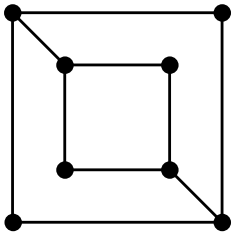
Due Friday, October 24

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

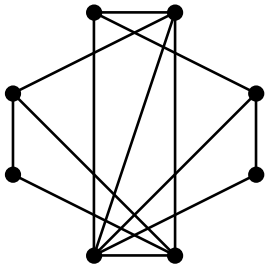
1. Determine the number of labeled spanning trees of the Petersen graph. (*Hint:* Use the Matrix Tree Theorem, Theorem 10.3 in Edition 4 and Theorem 3.5 in Edition 5. I recommend using a computer to compute the determinant.)

2. A graph is *outerplanar* if it has a planar drawing with all the vertices lying on the outside face. Use the Four Color Theorem to show that the chromatic number of an outerplanar graph is at most three. (*Hint:* Add a vertex to the outside face with edges to all the other vertices.)

3. (a) Use Kuratowski's Theorem to show that $K_{2,3}$ and K_4 are not outerplanar.
   (b) Use Kuratowski's Theorem to show that a graph is outerplanar if and only if it does not contain a subgraph homeomorphic to $K_{2,3}$ or K_4.
   (c) For each of the following graphs, determine if the graph is outerplanar. If the graph is outerplanar, show a planar drawing with all the vertices lying on the outside face. If the graph is not outerplanar, show that it contains a subgraph homeomorphic to $K_{2,3}$ or K_4.



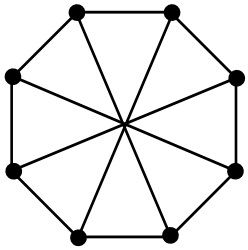
Graph 1



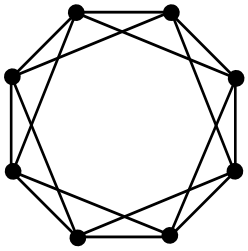
Graph 2

4. A graph has *genus* g if the graph can be drawn on a g-holed-torus without edge crossings but cannot be drawn on a $(g-1)$-holed-torus without edge crossings. The genus of a graph G is denoted $g(G)$.
   (a) Determine the genus of $K_{3,5}$.
   (b) Show that $g(K_{r,s}) \geq \frac{1}{4}(r-2)(s-2)$. (*Hint:* Recall that for graphs drawn on a g-holed torus, $v - e + f = 2 - 2g$.)

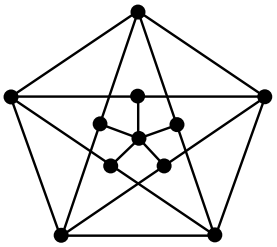
5. For each of the following graphs, determine the chromatic number of the graph, and show a coloring that uses the minimum number of colors. (You do not need to prove that your answer is a minimum.)



Graph 1



Graph 2



Graph 3