# Math 317 Homework 8
Due Friday, November 7

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. For each of the following graphs, compute the chromatic index $\chi'(G)$ of the graph, and show an edge-coloring that uses the minimum number of colors.
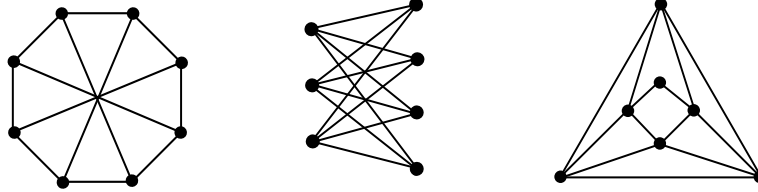
2. (a) Prove that the chromatic polynomial of $K_{2,s}$ is
$$k(k-1)^s + k(k-1)(k-2)^s$$

   (b) Prove that the chromatic polynomial of C_n is
$$(k-1)^n + (-1)^n(k-1)$$

3. Tom goes for a run every morning. When he leaves his house for his run he is equally likely to go out either the front or the back door; and similarly, when he returns he is equally likely to go to either the front or back door. He owns 5 pairs of running shoes, which he takes off after the run at whichever door he happens to be. If there are no shoes at the door from which he leaves to go running he runs barefooted.

   (a) Model this situation with a Markov chain. (You will need to decide what the states of the Markov chain are.) Write down the transition matrix and associated digraph.

   (b) Is this Markov chain ergodic? Explain your answer.

   (c) In the long run, what proportion of days does he run barefooted? Justify your answer.

4. Suppose that there exists a polynomial-time algorithm to determine whether a graph is 4-colorable. Show that there exists a polynomial time algorithm to determine whether a graph is 3-colorable. (This shows that if 3-colorability is NP-complete, then 4-colorability is NP-complete also.)