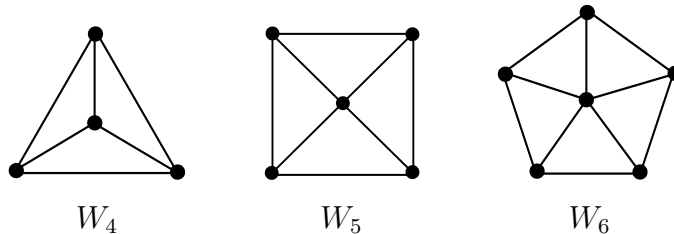


# Math 317 Homework 9

Due Friday, November 14

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Let  $G$  be a simple graph that is not a null graph. Prove that the sum of the coefficients of  $P_G(k)$  is 0. (*Hint*: When a function is a polynomial, how can one obtain the sum of the coefficients?)
2. The wheel graph  $W_n$  is the  $n$ -vertex graph consisting of a cycle with  $n - 1$  vertices and an additional vertex that is adjacent to all of the vertices in the cycle. The wheel graphs  $W_4$ ,  $W_5$ , and  $W_6$  are shown below:



Determine the chromatic polynomial of  $W_n$  for  $n \geq 4$ . Prove your answer.

3. An **infinite graph** is a graph  $G$  where both the vertex set  $V(G)$  and the edge set  $E(G)$  are infinite. (In Edition 4 of the textbook, see Section 16 of the textbok for more information about infinite graphs. In Edition 5 of the textbook, see pages 24, 38, and 44.)
  - (a) Find an Eulerian trail in the infinite square lattice. (The infinite square lattice is Figure 16.1 in Edition 4 and Figure 1.45 in Edition 5 of the textbook.)
  - (b) Give an example of a connected infinite graph in which every vertex has even degree, but the graph is not Eulerian.