# Math 317 Midterm

Due Friday, November 2 by 5pm

You must work completely on your own, consulting only the textbook, your course notes, and your homeworks as references. Show all of your work. If you have questions, you can come to my office hours or ask me via e-mail. You should write your solutions neatly and legibly. Good luck!

1. [**36 pts.**] Let G be a simple graph with edges $e_1, e_2, \ldots, e_m$. The *line graph* $L(G)$ is the simple graph with vertices $v_1, v_2, \ldots, v_m$ such that v_i is adjacent to v_j if and only if the edges e_i and e_j are adjacent in G.

   (a) Draw the line graph of K_4 and the line graph of $K_{2,3}$.

   (b) Consider the following graph G:
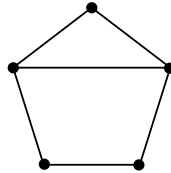
   Does there exist a simple graph H such that $L(H) = G$? If so, what is H?

   (c) Determine the number of cycles of length 3 in the line graph of the Petersen graph.

   (d) Let G be a simple graph with degree sequence $(d_1, d_2, \ldots, d_n)$. Find a formula for the number of edges in $L(G)$ in terms of $d_1, d_2, \ldots d_n$.

2. [**20 pts.**] The following directed graph represents a city, with each edge of the graph corresponding to a street of the city. All of the streets are one-way streets in the indicated direction. After a snowstorm, a snow removal truck needs to start at the garage, drive down every street plowing snow, and end back at the garage.
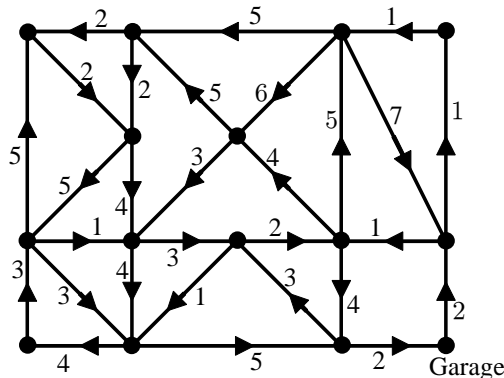
   The edges are labeled with the length of time it takes to drive down the given street. Since the graph does not have an Eulerian trail, the truck will need to drive down some streets more than once. Determine the minimum amount of time required to plow all of the streets. Justify your answer.

3. **[24 pts.]** Let G be a simple planar graph with no cycles of length 3.

   (a) Use Euler's formula to prove that G contains a vertex with degree 3 or less.

   (b) Use part (a) to prove that G is 4-colorable without using the Four Color Theorem.

4. **[20 pts.]** Let G be a simple graph with n vertices, and let $\bar{G}$ be the complement of G. (The complement of G is defined on page 20 of Edition 4 and page 14 of Edition 5 of the textbook.) Use induction to prove that $\chi(G) + \chi(\bar{G}) \leq n + 1$.