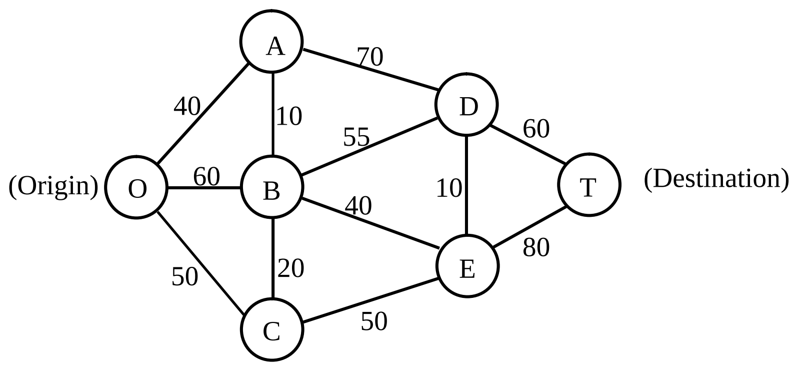
Math 322: Answers to Practice Problems for Final

<u>Edition 9:</u>
The answers to the starred problems are in the back of the book. Here are the answers to the non-starred problems:

9.3–2. (a) We create a graph with each town as a vertex and an edge between towns that are connected by a road.



(b) The shortest path is: $O \to A \to B \to E \to D \to T$. It has length 160.

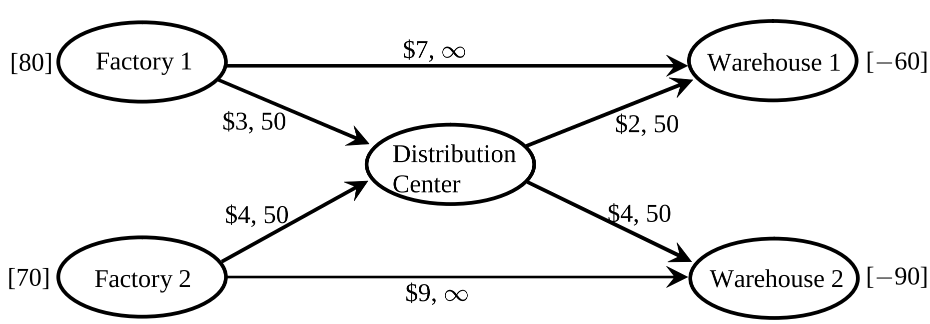
9.3–4. (a) There are two shortest paths:

- $O \to A \to B \to D \to T$
- $O \to A \to B \to E \to D \to T$

Both paths have length 16.

(b) The shortest path is $O \to C \to F \to G \to T$. It has length 17.

9.6–3. (a) The network representation is:



On each edge, the first number is the cost and the second number is the capacity.

(b) The variables for the linear program are:

$$
\begin{aligned}
x_{F1,W1} &= \text{amount shipped from Factory 1 to Warehouse 1} \\
x_{F1,D} &= \text{amount shipped from Factory 1 to the Distribution Center} \\
x_{F2,W2} &= \text{amount shipped from Factory 2 to Warehouse 2} \\
x_{F2,D} &= \text{amount shipped from Factory 2 to the Distribution Center} \\
x_{D,W1} &= \text{amount shipped from the Distribution Center to Warehouse 1} \\
x_{D,W2} &= \text{amount shipped from the Distribution Center to Warehouse 2}
\end{aligned}
$$

The the following linear program will determine the optimal shipment pattern so as to minimize the total cost:

$$
\begin{aligned}
&\text{Min } Z = 7x_{F1,W1} + 3x_{F1,D} + 9x_{F2,W2} + 4x_{F2,D} + 2x_{D,W1} + 4x_{D,W2} \\
&\text{subject to} \\
&x_{F1,D} + x_{F1,W1} = 80 \\
&x_{F2,D} + x_{F2,W2} = 70 \\
&x_{F1,W1} + x_{D,W1} = 60 \\
&x_{D,W2} + x_{F2,W2} = 90 \\
&x_{D,W1} + x_{D,W2} - x_{F1,D} - x_{F2,D} = 0 \\
&x_{F1,D} \leq 50, x_{F2,D} \leq 50, x_{D,W1} \leq 50, x_{D,W2} \leq 50 \\
&x_{F1,W1}, x_{F1,D}, x_{F2,W2}, x_{F2,D}, x_{D,W1}, x_{D,W2} \geq 0
\end{aligned}
$$

11.3–1. (a) Let x_i = amount of product i produced. And let

$$
y_i = \begin{cases} 0 & \text{if product } i \text{ is not produced} \\ 1 & \text{if product } i \text{ is produced} \end{cases}
$$

To change condition 2 into inequalities, note that condition 2 is equivalent to:

$$\text{If } y_3 + y_4 \geq 1 \text{ then } y_1 + y_2 \geq 1.$$

This is equivalent to:

$$\text{Either } y_3 + y_4 \leq 0 \text{ or } y_1 + y_2 \geq 1.$$

In addition to the variables listed above, there will also be two more binary variables y_5 and y_6 based on conditions 2 and 3. The integer program is:

$$\begin{aligned}
&\text{Max } 70x_1 + 60x_2 + 90x_3 + 80x_4 - 50{,}000y_1 - 40{,}000y_2 - 70{,}000y_3 - 60{,}000y_4 \\
&\text{subject to} \\
&x_1 \le My_1,\ x_2 \le My_2,\ x_3 \le My_3,\ x_4 \le My_4 \\
&y_1 + y_2 + y_3 + y_4 \le 2 \\
&-y_1 - y_2 \le -1 + My_5 \\
&y_3 + y_4 \le M(1 - y_5) \\
&5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6000 + My_6 \\
&4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6000 + M(1 - y_6) \\
&x_1, x_2, x_3, x_4 \ge 0 \\
&y_1, y_2, y_3, y_4, y_5, y_6 \text{ are binary}
\end{aligned}$$

The value of M is a large number. For example, $M = 50{,}000$ works.

11.3–7. (a) Let x_i = the number of planes produced for customer i. Define y_1 and y_2 as follows:

$$y_1 = \begin{cases} 0 & \text{no airplanes are produced for customer 1} \\ 1 & \text{if airplanes are produced for customer 1} \end{cases}$$

$$y_2 = \begin{cases} 0 & \text{no airplanes are produced for customer 2} \\ 1 & \text{if airplanes are produced for customer 2} \end{cases}$$

Then, the integer program is:

$$\begin{aligned}
&\text{Max } 2x_1 + 3x_2 + 0.8x_3 - 3y_1 - 2y_2 \\
&\text{subject to} \\
&x_1 \le My_1,\ x_2 \le My_2 \\
&0.2x_1 + 0.4x_2 + 0.2x_3 \le 1 \\
&x_1 \le 3, x_2 \le 2, x_3 \le 5 \\
&x_1, x_2, x_3 \ge 0 \\
&x_1, x_2, x_3 \text{ are integer} \\
&y_1, y_2 \text{ are binary}
\end{aligned}$$

The value of M is a large number. For example, $M = 5$ works.

12.5–1. (a) The sequence of points obtained using the gradient search is: $(1, 1) \to (1, 3/4) \to (3/4, 3/4) \to (3/4, 5/8)$. At the point $(3/4, 5/8)$, the gradient is $(-1/4, 0)$. Since the absolute value of both partial derivatives is ≤ 0.25, we can stop here.

Thus, the answer is: $\mathbf{x}^* \approx (3/4, 5/8)$.

(b) $\mathbf{x}^* = (1/2, 1/2)$