Math 322 Homework 1

Due Friday, February 9 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. A baker has 150 units of flour, 90 units of sugar, and 150 units of raisins. A loaf of raisin bread requires 1 unit of flour, 1 unit of sugar, and 2 units of raisins. A raisin cake requires 5 units of flour, 2 units of sugar, and 1 unit of raisins. The baker makes a profit of \$1.75 for each loaf of raisin bread he sells and a profit of \$4.00 for each raisin cake.

   (a) Set up a linear program to determine how much raisin bread and raisin cake the baker should make in order to maximize profit.

   (b) Graph the feasible region for the linear program, and determine the coordinates of the corner points.

   (c) How many loaves of raisin bread and how many raisin cakes should the baker make in order to maximize profit.

2. Consider the following linear programming problem, in which G is a constant:

$$\begin{aligned} &\text{Min } z = x + y \\ &\text{subject to} \\ &x - y \leq 1 \\ &2x + y \geq G \\ &y \leq 6 \\ &x, y \geq 0 \end{aligned}$$

   Let $Z(G)$ equal the optimal value of the linear programming problem for a given value of G.

   (a) Plot the feasible region if $G = 10$, and determine the coordinates of the vertices. Use this to find $Z(10)$.

   (b) For what values of G are there feasible solutions (ie, for what values of G does $Z(G)$ exist)?

   (c) Determine $Z(G)$ for all values of G that give a feasible region, and write your answer as a piecewise function.

   (d) Determine $Z'(G)$ for all values of G for which the derivative is defined. (The derivative tells us how quickly the optimal value changes as G changes.)

3. An airline wants to assign telephone operators round the clock. The operators are willing to work for eight to twelve hours every day of the week. The airline breaks up each day into four 6-hour periods. They have decided how many operators are needed during each four hour period as shown below.

Time Period	**Operators Needed**
12am to 4am	25
4am to 8am	35
8am to 12pm	50
12pm to 4pm	60
4pm to 8pm	50
8pm to 12am	40

Operators can be hired to work 8 or 12 hour consecutive shifts. Each operator works the same shift seven days a week with no days off. Operators are paid $10.00 per hour for the first 8 hours they work and $14.00 per hour for the extra four if they work the longer length shift.

(a) Assuming that fractional numbers of operators are allowed, formulate a linear program that will minimize the total labor costs for the week, while satisfying the total demand. Note that the operators are permitted to start at 12am, 4am, 8am, 12pm, 4pm, or 8pm. An operator starting at 8pm would end his or her shift on the following day. (*Hint:* There should only be six constraints, not counting non-negativity constraints.)

(b) Use Excel to solve the linear program from part (a). How many operators should they hire, what shifts should those operators work, and what is the total cost?

4. Two friends Susan and Alice have decided to go on a camping trip. They can choose between n different types of food to bring with them. One unit of food type i weighs a_i pounds and has a utility of c_i. Together, Susan and Alice can carry at most b pounds of food in their backpacks.

(a) Assuming that fractional items are allowed, formulate a linear programming problem that will determine how many units of each food type they should bring in order to maximize the utility of their backpack.

(b) Suppose that $n = 2$. Graph the feasible region in this case, and determine the coordinates of the vertices. Determine the possible optimal solutions to the linear programming problem, and explain how you would determine which of these is optimal.

(c) Suppose that $n = 3$. Graph the feasible region in this case, and determine the coordinates of the vertices. Determine the possible optimal solutions to the linear programming problem, and explain how you would determine which of these is optimal.

(d) What is the optimal solution to the general problem (where there are n food types)?

(e) Under what conditions would multiple optimal solutions exist?