Math 322 Homework 10

Due Friday, April 27 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following BIP:

$$\begin{aligned}
&\text{Max } Z = 3x_1 + 3x_2 + 5x_3 - 2x_4 - x_5 \\
&\text{subject to} \\
&x_1 + 2x_2 - 3x_4 - x_5 \leq 0 \\
&-15x_1 + 30x_2 - 35x_3 + 45x_4 + 45x_5 \geq 50 \\
&x_1, x_2, x_3, x_4, x_5 \geq 0 \\
&x_1, x_2, x_3, x_4, x_5 \text{ are binary}
\end{aligned}$$

   Use the BIP branch-and-bound algorithm to solve this BIP. You can use Excel to solve the LP's that occur during the algorithm. Show the branching tree that results when you perform the algorithm, and clearly label the vertices of the tree with the solution to the corresponding LP and the resulting bound for Z.

2. Consider the following MIP:

$$\begin{aligned}
&\text{Min } Z = 5x_1 + x_2 + x_3 + 2x_4 + 3x_5 \\
&\text{subject to} \\
&x_2 - 5x_3 + x_4 + 2x_5 \geq -2 \\
&5x_1 - x_2 + x_5 \geq 7 \\
&x_1 + x_2 + 6x_3 + x_4 \geq 4 \\
&x_1, x_2, x_3, x_4, x_5 \geq 0 \\
&x_1, x_2, x_3 \text{ are integers}
\end{aligned}$$

   Use the MIP branch-and-bound algorithm to solve this MIP. You can use Excel to solve the LP's that occur during the algorithm. Show the branching tree that results when you perform the algorithm, and clearly label the vertices of the tree with the solution to the corresponding LP and the resulting bound for Z.

3. Consider the following integer program:

$$\text{Max } Z = 10x_1 + 22x_2 + 5x_3 + 15x_4 + 17x_5 + 12x_6 + 4x_7$$

subject to

$$5x_1 + 3x_2 + 8x_3 + 9x_4 + 16x_5 + 5x_6 + 10x_7 \leq 700$$

x_1, x_2, x_3 binary

x_4, x_5, x_6, x_7 integer

$0 \leq x_4, x_5, x_6, x_7 \leq 200$

For each of the parts below, explain how to modify the integer program, so that the given conidtion is satisfied is also satisfied. Each of the parts is independent, so that no part depends on the parts preceding it.

(a) Modify the integer program so that at least two of the following constraints are satisfied: (i) $x_4 \geq 50$, (ii) $x_5 \leq 25$, or (iii) $x_6 + x_7 \leq 100$.

(b) Modify the integer program so that x_5 equals 9, 15, or 20.

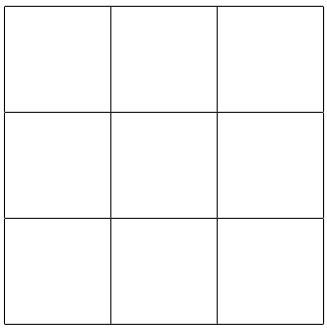
(c) Modify the integer program so that $2x_4 + x_5 \leq 50$ or $4x_4 - x_5 \geq 20$, but not both.

(d) Modify the integer program so that if $x_2 = 1$, then $x_1 = 0$.

(e) Modify the integer program so that x_1, x_2, and x_3 cannot all equal 1.

(f) Modify the integer program so that x_7 is divisible by 3 but not by 6. (Note that this means that the remainder when dividing by 6 is 3.)

4. Consider a 3×3 game board:



You are required to fill each square with a number between 1 and 9 such that the sum of the numbers in each row, each column, and each diagonal equals 15. Additionally, the numbers in all the squares must be distinct.

(a) Formulate an integer program to determine an assignment of numbers to squares. (Since you are just trying to find a solution, your objective function can be anything.)

(b) Use Excel to solve the program. What is the solution? (When you enter the program in Excel, you can specify in Solver that the variables need to be AllDifferent, which will simplify solving the linear program in Excel. However, your answer to part (a) cannot simply say that the variables are all different; you will need to find a way to specify that as part of the integer program.)