

# Math 322 Homework 11

Due Monday, May 7 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following optimization problem:

$$\text{Minimize } f(x) = x^4 + x^2 - 4x$$

- (a) Show that the function  $f$  is convex (and thus has at most one local minimum).
  - (b) Does the optimal solution lie in the interval  $0 \leq x \leq 1$  or the interval  $1 \leq x \leq 2$ ? Explain your answer.
  - (c) Use the bisection method with the bounds from part (b) and with error tolerance  $\epsilon = 0.01$  to approximately solve the optimization problem. For each iteration, show the resulting lower bound  $\underline{x}$  and upper bound  $\bar{x}$ . What is the approximate value for  $x$  and  $f(x)$  at the optimal solution? (You may want to use Excel to help with the computations involved in the bisection method.)
  - (d) Use Newton's method with starting value  $x_1 = 1$  and with error tolerance  $\epsilon = 10^{-6}$  to approximately solve the optimization problem. Show the resulting  $x_i$  for each iteration. What is the approximate value for  $x$  and  $f(x)$  at the optimal solution? (You may want to use Excel to help with the computations involved in Newton's method.)
2. Consider the following function:

$$f(x, y) = 3ye^x - e^{3x} - y^3$$

- (a) This function has one critical point. Find it.
  - (b) Compute the Hessian of  $f$  and use it to show that the critical point is a local maximum.
  - (c) Show that  $f$  does not have a global maximum by considering how  $f$  behaves along the  $y$ -axis.
3. Consider the following function:

$$f(x, y, z) = x^2 + y^2 + z^2 - xy - yz + x + z$$

- (a) This function has one critical point. Find it.
- (b) Compute the Hessian of  $f$ , and use it to determine whether the critical point is a local max, local min, or neither?
- (c) Is the critical point a global max, global min, or neither? Justify your answer.

4. Consider the following optimization problem:

$$\text{Maximize } f(x, y) = 4x + 2y + x^2 - x^4 - 2xy - y^2$$

- (a) Complete one iteration of the gradient search procedure starting at  $(x, y) = (1, 0)$ . Use Newton's method with  $\epsilon = 0.01$  to find the maximum of the function in the direction of the gradient vector. (You may want to use Excel to help with the computations involved.)
- (b) Continue using the gradient search procedure using  $\epsilon = 0.05$  (still using Newton's method with  $\epsilon = 0.01$  to maximize the function in the direction of the gradient vector). What are the approximate values of  $x$  and  $y$  at the optimal solution? What is the approximate value of  $f$  at the optimal solution?