Math 322 Homework 2

Due Friday, February 16 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. For each of the following Linear Programming problems:

   - Sketch a graph of the feasible region.
   - Determine the coordinates of the vertices.
   - Determine the optimal solution(s) to the Linear Programming problem.

(a)
$$\begin{aligned} &\text{Max } 9x + 6y + 6z \\ &\text{subject to} \\ &2x + 3y + 4z \leq 24 \\ &6x + 4y + 4z \leq 36 \\ &x, y, z \geq 0 \end{aligned}$$

(b)
$$\begin{aligned} &\text{Min } 2x + y + 3z \\ &\text{subject to} \\ &5x + 2y + 7z = 420 \\ &3x + 2y + 5z \geq 280 \\ &x, y, z \geq 0 \end{aligned}$$

2. Consider the following linear program, in which A and B are constants:

$$\begin{aligned} &\text{Min } Ax + By \\ &\text{subject to} \\ &x - 2y \leq 4 \\ &x - y \geq -2 \\ &x, y \geq 0 \end{aligned}$$

   (a) Sketch the graph of the feasible region, and determine the coordinates of the vertices.

   (b) Give values for A and B so that the point $(2, 0)$ is an optimal solution, but $(1, 1)$ is not optimal.

   (c) Give values for A and B so that the optimal solution is unbounded.

(d) Give values for A and B so that the point $(2, 4)$ is an optimal solution, but $(1, 1)$ is not optimal.

(e) Add a constraint to the problem that makes the problem infeasible.

3. *In class, we considered the problem of where to locate a business that sold crabs to restaurants, where the restaurants were located along a 10 mile line segment. In this problem, we will suppose instead that the restaurants are located along a square grid that has a length of 10 miles and a width of 10 miles.*

Two people have decided to go into business together selling crabs that they catch along the Baltimore coast line After they catch the crabs, they bring them back to their processing plant where they are cleaned, clawed, and wrapped to go. Following this process, they ship the crabs to n different seafood restaurants located in a 10 mile $\times$ 10 mile square. Each restaurant i has two coordinates (a_{i1}, a_{i2}) and demands d_i crabs.

We will measure the distance between the processing plant and the restaurant using the "Manhattan metric" (which is sometimes called the "taxicab metric"). That is, all streets go either North-South or East-West, so the distance from location (x_i, y_i) to (x_j, y_j) is $|x_i - x_j| + |y_i - y_j|$. The unit transportation cost of shipping a crab is $\$c$ per mile (so it costs $\$mc$ per mile to ship m crabs).

Their goal is to locate the processing plant at a point (x, y) inside the 10×10 square to minimize the total transportation costs.

(a) Formulate the problem of determining where the processing plant should be located as a mathematical program to meet all demands while minimizing the transportation costs. (Your answer to this part does not need to be a linear program.)

(b) Convert the mathematical program of part (a) to a linear programming problem.

(c) It is claimed that there is always an optimal solution to the linear programming problem in which the processing plant is located at one of the restaurants. Show that this claim is false.

(d) It is claimed that there is always an optimal solution to the linear programming problem in which the processing plant is located at the x-coordinate of one restaurant and the y-coordinate of one restaurant (they do not need to be the same restaurant). Show that this claim is true.