Math 322 Homework 3

Due Friday, February 23 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Willy Wonka's Candy Company produces three types of candy:

   - Wonka Bars
   - Bottle Caps
   - Giant Sweet Tarts

   In order to produce the different types of candy, Willy can run three different production processes as described below. Each process involves blending different types of sugars in the Magical Factories Mixer.

Process 1:	Running Process 1 for one hour: Costs: \$5 Requires: Two barrels of **sugar type A** and three barrels of **sugar type B** Output: Two Wonka Bars and one packet of Bottle Caps

Process 2:	Running Process 2 for one hour: Costs: \$4 Requires: One barrel of **sugar type A** and three barrels of **sugar type B** Output: Three packets of Bottle Caps

Process 3:	Running Process 3 for one hour: Costs: \$1 Requires: Three barrels of **sugar type A** and three packets of **Bottle Caps** Output: Two Wonka Bars and one packet of Giant Sweet Tarts

   Each week we can purchase:

   - 200 barrels of sugar type A at \$2 Per Barrel
   - 300 barrels of sugar type B at \$3 Per Barrel

   Assume that they can sell everything that they can produce.

   - Wonka Bars are sold at \$9 per bar.
   - Bottle Caps are sold at \$10 per packet.
   - Giant Sweet Tarts are sold at \$24 per packet.

   Assume that 100 hours of mixing time are available.

   (a) Formulate a Linear Programming Problem whose solution will maximize Willy Wonka's profits.

(b) Assume that instead of having 200 barrels of sugar type A and 300 barrels of sugar type B available that you can order a total of 500 barrels. Show how to modify your Linear Programming formulation in part (a) to account for this revised problem.

(c) Suppose that instead of selling three candies seperately, they can only be sold as part of a box consisting of one Wonka Bar, two packets of Bottle Caps, and one pack of Giant Sweet Tarts. Each Wonka Box sells for \$54. Modify your Linear Programming formulation in part (a) to model this new scenario. Assume that you again have 200 barrels of sugar type A and 300 barrels of sugar type B. (*Hint:* You may find it helpful to start by creating a mathematical program where the objective function is a minimum of linear functions, and then converting to a linear program.)

Note: You do not need to solve any of the linear programs in parts (a) through (c). You just need to set up the linear program. You should clearly state what each variable represents, and also clearly state the function to be minimized or maximized along with the constraints.

2. Suppose that the following tableau occurs while solving a maximization problem using the simplex algorithm, where A, B, C, D, E, F, G, and H are all constants:

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
1	A	0	0	B	C	D	0	E
0	F	0	1	1	1	3	0	4
0	-2	1	0	2	G	-1	0	2
0	-1	0	0	0	H	1	1	3

(a) What is the current CPF?

(b) For each of the following situations, give values for A through H that would make the given condition true.

i. The current CPF is a unique optimal solution.

ii. The current CPF is not optimal, the only candidate for entering the basis is s_1, and when the next iteration is performed, x_3 will leave the basis.

iii. The current CPF is not optimal, the only candidate for entering the basis is x_1, and the problem is unbounded from above.

iv. The current CPF is optimal, and there exists exactly one other CPF that is also optimal.

v. The current CPF is optimal, and there also exists an extreme ray of alternative optimal solutions.

3. Consider the following Linear Programming Problem:

$$\begin{aligned}
&\text{Max } 50x_1 + 25x_2 + 20x_3 + 40x_4 \\
&\text{subject to} \\
&2x_1 + x_2 \leq 30 \\
&x_3 + 2x_4 \leq 20 \\
&x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

Work through the simplex algorithm step by step to find *all* of the optimal solutions. Show the steps of the simplex algorithm — you do not need to write out every row operation, but you should write the tableau for each iteration and indicate which variable is entering the basis and which variable is leaving the basis.