Math 322 Homework 4
Due Friday, March 2 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. A toy company produces three types of toys — trains, trucks, and cars. They have three different machines that are used to make the toys: Machine A, Machine B, and Machine C. Creating a toy train requires 1 minute on Machine A, 3 minutes on Machine B, and 1 minute on Machine C. Creating a toy truck requires 2 minutes on Machine A and 4 minutes on Machine C. Creating a toy car requires 1 minute on Machine A and 2 minutes on Machine B. Machine A can be used for total of 430 minutes each day; machine B can be used for a total of 460 minutes each day; and machine C can be used for a total of 420 minutes each day. The revenue for creating a train is \$3; the revenue for a truck is \$2; and the revenue for a car is \$5.

   (a) Formulate a linear programming problem to maximize the revenue for the toy company.

   (b) Use the simplex algorithm to solve the linear programming problem. Show the steps of the simplex algorithm — you do not need to write out every row operation, but you should write the tableau for each iteration and indicate which variable is entering the basis and which variable is leaving the basis.

2. Consider the following linear program:

$$\text{Min } 2x - 3y$$
$$\text{subject to}$$
$$x + 2y \leq 12$$
$$2x + y \leq 8$$
$$-x + y \leq 12$$
$$x \geq -5$$
$$(\text{no lower bound constraint for } y)$$

   (a) Graph the feasible region, and determine the coordinates of the vertices.

   (b) Work through the simplex algorithm step by step to find the optimal solution(s). Show the steps of the simplex algorithm — you do not need to write out every row operation, but you should write the tableau for each iteration and indicate which variable is entering the basis and which variable is leaving the basis.

   (c) Identify the sequence of CPF solutions that the simplex algorithm uses to find the optimal solution.

3. Consider the following linear program:

$$\text{Max } 2x + 5y + 3z$$
$$\text{subject to}$$
$$x - 2y + z \geq 20$$
$$2x + 4y + z = 50$$
$$x, y, z \geq 0$$

Work through the simplex algorithm step by step to find the optimal solution(s). Show the steps of the simplex algorithm — you do not need to write out every row operation, but you should write the tableau for each iteration and indicate which variable is entering the basis and which variable is leaving the basis.

4. Consider the three-dimensional polytope shown below:
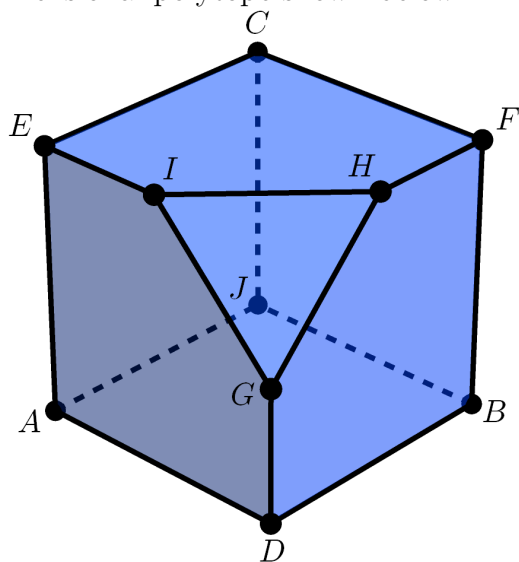
The point J lies at the origin, and the points A, B, and C lie on the x-, y- and z-axes, respectively. The coordinates of the vertices are given in the following table:

A	$(1,0,0)$	F	$(0,1,1)$
B	$(0,1,0)$	G	$(1,1,1/2)$
C	$(0,0,1)$	H	$(1/2,1,1)$
D	$(1,1,0)$	I	$(1,1/2,1)$
E	$(1,0,1)$	J	$(0,0,0)$

Suppose that this polytope is the feasible region for a linear programming problem.

(a) Suppose that the simplex algorithm for the linear programming problem starts at J and that the optimal solution occurs at H. For each of the following paths of vertices, state whether they could be legitimate paths for the simplex algorithm. If it is not a possible path for the simplex algorithm, explain why not.

    i. $J \to B \to F \to H$

    ii. $J \to D \to G \to H$

    iii. $J \to A \to B \to F \to H$

    iv. $J \to A \to D \to B \to J \to C \to F \to H$

(b) Find an objective function $Z = Ax + By + Cz$ so that the points A, B, D, and J are all maximal solutions, but the point $(0, 0, 1)$ is not maximal.

(c) Find an objective function $Z = Ax + By + Cz$ so that the points G, H, and I are all maximal solutions, but the point $(0, 0, 1)$ is not maximal.

(d) Determine the inequalities that describe the feasible region.