

Math 322 Homework 5

Due Friday, March 9 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following linear program:

$$\text{Max } 3x_1 + 2x_2 + 5x_3$$

subject to

$$3x_1 - x_2 - x_3 \leq -6$$

$$2x_1 - x_2 - x_3 \geq 3$$

$$2x_1 - 3x_2 + x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

Work through the simplex algorithm step by step to show that there are no feasible solutions to this linear program.

2. Recall that during the simplex algorithm a *degenerate basic variable* is a basic variable that is equal to 0.
 - (a) Give an example of a tableau for which the following holds:
 - There is a degenerate basic variable.
 - There is a nonbasic variable in the Z row that has a zero coefficient.
 - The current CPF is the unique optimal solution (but there may be more than one optimal tableau).
 - (b) Give an example of a tableau for which the following holds:
 - There is a degenerate basic variable.
 - There is a nonbasic variable in the Z row that has a zero coefficient.
 - The linear program has multiple optimal solutions, but only one optimal CPF.

3. The Gemstone Tool company produces wrenches and pliers. They are both made from steel, and the process involves molding on a Molding Machine, and assembling on an Assembly Machine. The amount of steel used and the daily availability of steel are shown in the following table. The table also shows the amount of machine time needed for each product as well as the availability of machine time.

	Wrenches	Pliers	Availability
Steel (lbs.)	1.5	1.0	27,000 lbs./day
Molding Machine (hrs.)	1.0	1.0	21,000 hours/day
Assembly Machine (hrs.)	0.3	0.5	9,000 hours/day

There is demand for at most 16,000 wrenches and for at most 15,000 pliers. For every 1000 wrenches that are sold, the company makes \$100; for every 1000 pliers which are sold, the company makes \$130.

- (a) Formulate a linear program to determine the optimal level of production for wrenches and pliers that maximizes the earnings.
- (b) Solve the problem using Excel. What is the optimal solution to the linear program? What is the optimal objective value?
- (c) Use Excel to create the Sensitivity Report. Print out the Sensitivity Report, and include it with your homework. Use the sensitivity report to answer the following questions:
 - i. Suppose the amount of steel available increases to 28,000 lbs./day. What is the new optimal objective value?
 - ii. Suppose the Molding Machine changes to only being available for 20,000 hours/day. What is the new optimal objective value?
 - iii. Suppose that the demand for pliers increases to 18,000 pliers. What is the new optimal objective value?
 - iv. The company could purchase additional Assembly Machines to increase the amount of Assembly Machine hours available each day. How much should they be willing to spend to add an additional 100 hours of Assembly Machine time?
 - v. Suppose that the price of wrenches increases, and that as a result, for every 1000 wrenches sold, the company makes \$110 (instead of \$100). Does this change the optimal solution? What is the new optimal objective value?
 - vi. Suppose that the demand for wrenches decreases to A wrenches. For what values of A is the current solution optimal?
- (d) Suppose that the company institutes a new policy — they must produce at least as many wrenches as pliers. Does the optimal solution change? Why or why not?

4. Consider the following linear programming problem:

$$\begin{aligned} & \text{Max } 5x_1 + 3x_2 \\ & \text{subject to} \\ & 2x_1 + 3x_2 \leq 12 \\ & 2x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- (a) Sketch the graph of the feasible region for this linear programming problem, and determine the coordinates of the vertices.
- (b) Create the dual linear programming problem.
- (c) Sketch a graph of the feasible region for the dual problem, and determine the coordinates of the vertices.
- (d) Use the simplex algorithm to solve the primal problem (that is, the original problem, not the dual problem).
- (e) Identify the sequence of CPF solutions that the simplex algorithm uses to find the optimal solution.
- (f) Use the final tableau from part (d) to determine the optimal solution to the dual problem.
- (g) Each iteration of the simplex algorithm also corresponds to a point in the dual problem (although the point in the dual problem will not be a feasible solution until the final tableau). At each iteration, the coefficients of the slack variables in the top row of the tableau correspond to a point in the dual problem. Identify this sequence of points, and mark these points on your graph in part (c).