Math 322 Homework 7

Due Friday, April 6 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Consider the following zero-sum game:

		Player 2		
		A	B	C
Player 1	A	5	4	2
	B	1	3	4

   (a) Use the graphical method to determine Player 1's optimal mixed strategy according to the minimax criterion.

   (b) What is Player 2's optimal mixed strategy?

   (c) When both players use their optimal mixed strategy, what is Player 1's expected payoff? What is Player 2's expected payoff?

2. Consider the following zero-sum game:

		Player 2		
		A	B	C
Player 1	A	-2	1	2
	B	2	-1	0
	C	1	0	-2

   (a) If Player 2 knows that Player 1 is using the strategy $\frac{1}{2}A + \frac{1}{2}C$, what strategy should Player 2 use? What will Player 1's expected payoff be in this case?

   (b) Formulate a linear program to find Player 1's optimal mixed strategy according to the minimax criterion.

   (c) Formulate a linear program to find Player 2's optimal mixed strategy according to the minimax criterion.

   (d) Solve the linear program from part (b). What is Player 1's optimal mixed strategy? (You can use any method you want, including Excel, but you should give your answers as fractions rather than decimal approximations.)

   (e) Solve the linear program from part (c). What is Player 2's optimal mixed strategy? (You can use any method you want, including Excel, but you should give your answers as fractions rather than decimal approximations.)

   (f) Suppose that Player 1 and Player 2 both use their optimal strategy. What is Player 1's expected payoff? What is Player 2's expected payoff?

3. Consider the following zero-sum game, in which a, b, c, and d are constants:

		Player 2	
		Strategy 1	Strategy 2
Player 1	Strategy 1	a	b
	Strategy 2	c	d

Suppose that a and d are larger than b and c (this implies that there are no rows or columns that dominate, so that there is not a pure strategy optimal strategy).

(a) Determine Player 1's optimal mixed strategy in terms of a, b, c, and d.

(b) Determine Player 2's optimal mixed strategy in terms of a, b, c, and d.

(c) If both players use their optimal strategy, what is Player 1's expected payoff in terms of a, b, c, and d.

4. Consider the following **non-zero-sum** game (in each entry, the first number is Player 1's payoff and the second number is Player 2's):

		Player 2				
		A	B	C	D	E
Player 1	A	$(2, -1)$	$(2, 0)$	$(-5, -1)$	$(0, 0)$	$(1, -1)$
	B	$(2, 2)$	$(3, 3)$	$(-4, 3)$	$(1, 3)$	$(1, -2)$
	C	$(1, -1)$	$(-2, 2)$	$(-5, 4)$	$(0, 4)$	$(-1, 3)$

Recall that a *Nash equilibrium* is a pair of strategies (one for Player 1 and one for Player 2) such that if both players are using those strategies, neither player can do better by changing to a different strategy.

(a) Is (Row A, Column B) a Nash equilibrium? Why or why not?

(b) Is (Row B, Column D) a Nash equilibrium? Why or why not?

(c) Find all of the pure strategy Nash equilibria?