Math 322 Homework 8

Due Friday, April 13 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. Suppose there are 6 bus drivers, and each bus driver needs to be assigned a morning route and an afternoon route. The morning routes have durations of 75 minutes, 100 minutes, 50 minutes, 125 minutes, 85 minutes, and 145 minutes. The afternoon routes have durations of 85 minutes, 95 minutes, 100 minutes, 150 minutes, 140 minutes, and 60 minutes. If a bus a bus drivers total time (morning plus afternoon) exceeds 200 minutes, he or she is paid overtime, which is $20 per hour). The objective is to assign each driver to a morning route and an afternoon route so as to minimize the total amount of overtime.

   (a) Formulate this problem as an Assignment Problem.

   (b) Use the Hungarian Algorithm to determine the optimal solution to the Assignment Problem. At the optimal solution, how many minutes of overtime do the bus drivers work?

2. A Christmas Toy Company wishes to ship toys from its 5 factories to 5 stores. Each factory will ship one truckload of toys to one store. Each store can only accept one truckload of toys. The following table shows the profit that the company will receive if it ships the toys from the given factory to the given store:

<table>
<tr><td></td><td colspan="6" align="center">Stores</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td></td><td>A</td><td>$500</td><td>$300</td><td>$700</td><td>$200</td><td>$400</td></tr>
<tr><td></td><td>B</td><td>$800</td><td>$200</td><td>$400</td><td>$300</td><td>$700</td></tr>
<tr><td>Factories</td><td>C</td><td>$500</td><td>$300</td><td>$700</td><td>$500</td><td>$800</td></tr>
<tr><td></td><td>D</td><td>$1,000</td><td>$100</td><td>$500</td><td>$200</td><td>$100</td></tr>
<tr><td></td><td>E</td><td>$600</td><td>$700</td><td>$800</td><td>$300</td><td>$400</td></tr>
</table>

   The company wishes to maximize the total profit.

   (a) Use the Hungarian Algorithm to determine which factories should ship to which stores so as to maximize the company's profit. What is the company's profit at this optimal solution?

(b) The dual problem is:

> The Grinch wishes to stop the toy company from selling these goods, so the Grinch will pay the company to not produce goods at some of the factories, and he will pay the company to not sell goods at some of the stores. He wants to make sure that the toy company does not sell any goods, so the total amount that he pays a given factory plus the total amount that he pays a given store, must be at least as much as the company could make by shipping goods from the factory to the store. The Grinch wishes to minimize the total amount that he spends.

Formulate this dual problem as a linear program.

(c) Solve the linear program from part (b) (using any method you want). How much should the Grinch pay each factory to stop the company from selling the toys? How much should the Grinch pay each store to stop the company from selling the toys? What is the total cost to the Grinch?

3. The Move-It Company has two plants producing forklift trucks that then are shipped to three distribution centers. The production costs are the same at the two plants, and the cost of shipping for each truck is shown below for each combination of plant and distribution center:

Distribution Center

		1	2	3
Plants	A	$800	$700	$400
	B	$600	$800	$500

A total of 60 forklift trucks are produced and shipped each week. Each plant can produce and ship any amount up to a maximum of 50 trucks per week, so there is considerable flexibility on how to divide the total production between the two plants so as to reduce shipping costs. However, each distribution center must receive exactly 20 trucks per week.

Management's objective is to determine how many forklift trucks should be produced at each plant, and then what the overall shipping pattern should be to minimize the total shipping cost.

(a) Formulate this problem as a Transportation Problem. (Note that in a Transportation Problem, the total amount supplied by a vertex must *equal* a constant, not be less than or equal to a constant.)

(b) Use Excel to solve the Transportation Problem from part (a). How many forklift trucks should be produced at each plant? What is the overall shipping pattern? What is the total cost?

(c) Suppose instead that the distribution centers can each receive between 10 and 30 forklift trucks per week. The total number of trucks shipped to the distribution centers must still equal 60 trucks per week.

    i. Formulate this new problem as a Transportation Problem.

    ii. Use Excel to solve this new Transportation Problem. How many forklift trucks should be produced at each plant? How many trucks should each distribution center receive? What is the overall shipping pattern? What is the total cost?

4. Tom and Carol are playing a game in which each round, the winning player scores either 2 or 3 points. After many rounds Tom has a score of 15. We would like to determine the fewest number of rounds that Tom could have won to have a score of 15.

(a) Formulate this problem as a shortest path problem. Draw the graph including the weights on the edges. Indicate the starting vertex and ending vertex.

(b) Determine the shortest path in the graph from part (a). What is the fewest number of rounds that Tom could have won to have a score of exactly 15.