

Math 322 Homework 9

Due Friday, April 20 by 5pm

Solutions should be written neatly and legibly. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. You should reference all of your sources, including your collaborators.

1. (a) Four people must cross a canyon at night on a fragile bridge. At most two people can be on the bridge at once. Crossing requires carrying a flashlight, and there is only one flashlight (which can cross only by being carried). Alone, the four people cross in 10, 5, 2, 1 minutes, respectively. When two cross together, they move at the speed of the slower person. In 18 minutes, a flash flood coming down the canyon will wash away the bridge. Can the four people get across in time? Explain why or why not.
(b) Explain how to formulate the above problem as a shortest path problem. You do not need to draw the entire graph, but you should state what the vertices represent, what the edges represent, which vertex is the starting vertex, and which vertex is the ending vertex.
2. A company sells teddy bears to both Six Flags New England and Six Flags Great Adventure. They base their operation out of Seattle Washington. The company has two production plants, one in San Mateo, CA and one in Bakersfield, CA, which each produce exactly 5000 teddy bears. The bears are sent from the production plant to an inspection facility, in either Dallas or Houston. From the inspection facility, bears are shipped to the theme parks, where New England requires 3000 teddy bears and Great Adventure needs 7000. The shipping cost per bear between cities is shown in the table below.

	Dallas	Houston	Six Flags New England	Six Flags Great Adventure
San Mateo	\$1	\$1		
Bakersfield	\$4	\$1		
Dallas			\$4	\$6
Houston			\$5	\$3

- (a) Formulate the problem of satisfying demand while minimizing cost as a Minimum Cost Flow Problem.
- (b) Write the Minimum Cost Flow Problem as a linear program.
- (c) Use Excel to solve the Minimum Cost Flow Problem. How should the teddy bears be shipped? What is the total cost at the optimal solution?
- (d) Suppose there is a cost for inspecting the teddy bears. The cost is \$1 per bear in Houston and \$2 per bear in Dallas. Modify your answer to part (a) to take into account this change. (You do NOT need to solve the new problem.)

- (e) Suppose the facility in Houston can inspect at most 2000 teddy bears; hence, only 2000 teddy bears can be sent to the plant. Modify your answer to part (a) to take into account this change. (You do NOT need to solve the new problem.)

3. Consider the following integer program:

$$\begin{aligned} &\text{Max } 2x + 5y \\ &\text{subject to} \\ &2x + 3y \leq 12 \\ &x + 3y \leq 8 \\ &x, y \geq 0 \\ &x, y \text{ are integers} \end{aligned}$$

- (a) Sketch a graph of the region. Determine the coordinates of the vertices for the corresponding linear program (including vertices with non-integer coordinates).
- (b) What is the optimal solution and optimal objective value to the linear program where the variables are not required to be integers?
- (c) List all of the integer-valued points that occur in the feasible region.
- (d) What is the optimal solution and optimal objective value for the integer program?
4. A company manufactures three products whose daily labor and raw material requirements are given in the following table:

Product	Required daily labor (hr/unit)	Required daily raw material (lb/unit)
1	3	4
2	4	3
3	5	6

The profits per unit of the three products are \$25, \$30, \$22, respectively. The company has two locations for locating its plant. The two locations differ primarily in their availability of labor and raw material, as shown in the following table:

Location	Available daily labor (hr)	Available daily raw material (lb)
1	100	100
2	90	120

- (a) Formulate an integer program to determine where they should locate the plant and how much of each product they should produce in order to maximize their profit.
- (b) Use Excel to solve the integer program. Where should they locate the plant? How much of each product should they produce?