Math 322: Practice Problems for Final

1. Use the Hungarian algorithm to solve the assignment problems having the following cost tables. For each, determine the assignment of applicants to jobs that minimizes the total cost.

   (a) Jobs

Applicants	**1**	**2**	**3**
A	\$10	\$8	\$7
B	\$7	\$6	\$4
C	\$3	\$2	\$1

   (b) Jobs

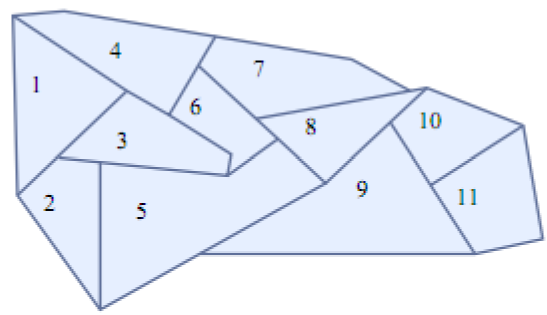
Applicants	**1**	**2**	**3**	**4**
A	\$5	\$8	\$6	\$7
B	\$9	\$5	\$7	\$8
C	\$5	\$9	\$8	\$4
D	\$6	\$3	\$5	\$9

2. United Airlines has five daily flights from Chicago to New York. The flights depart every 2 hours starting at 10 AM (with the last flight at 6 PM). The first three flights have a capacity of 100 passengers and the last two flights can accommodate 150 passengers each. If overbooking results in insufficient room for a passenger on a scheduled flight, United can divert a passenger to a later flight. It compensates any passenger delayed from his or her regularly scheduled departure by paying \$50 for every hour of delay. United can always accommodate passengers delayed beyond the 6 PM flight on the 9 PM flight of another airline that always has a great deal of spare capacity. The other airline charges United \$75 for each passenger that takes the 9 P.M. flight. Suppose that at the start of a particular day the five United flights have 110, 118, 103, 161, and 140 confirmed reservations.

   (a) Formulate the problem of determining the most economical passenger routing strategy as a Minimum Cost Flow problem.

   (b) Formulate the problem as a linear program.

3. Starbucks plans to open some coffee shops in a city that is composed of 11 neighborhoods in the following arrangement:



A Starbucks can be placed in any neighborhood. A Starbucks in any particular neighborhood is able to handle the customers for both its neighborhood and any adjacent neighborhoods (neighborhoods are adjacent if they meet along on edge; neighborhoods are not considered adjacent if they just meet at a point).

Formulate an integer program to determine the location of Starbucks coffee shops that minimizes the the total number of Starbucks while ensuring that every neighborhood either has a Starbucks or is adjacent to a neighborhood with a Starbucks.

4. A machine is used to produce two products (product A and product B). The daily capacity of the machine can produce at most 20 units of product A and 10 units of product B. Alternatively, the machine can be adjusted to produce at most 12 units of product A and 25 units of product B daily. Market analysis shows that the maximum daily demand for the two products combined is 35 units. The unit profits for the two respective products are \$10 and \$12. Formulate an integer program to determine how much of each product should be produced each day in order to maximize profit.

5. Consider the following BIP:

$$\begin{aligned} &\text{Max } Z = x_1 + x_2 + x_3 \\ &\text{subject to} \\ &3x_1 + 2x_2 + x_3 \leq 5 \\ &2x_1 + 3x_2 + 3x_3 \leq 7 \\ &x_1, x_2, x_3 \geq 0 \\ &x_1, x_2, x_3 \text{ are binary} \end{aligned}$$

If this problem is solved as a linear program (without the integer constraint), the solution is $x_1 = 0.8, x_2 = 0.8, x_3 = 1, Z = 2.6$.

Use Branch and Bound to solve the BIP. Show the branching tree that results when you perform the algorithm, and clearly label the vertices of the tree with the solution to the corresponding LP and the resulting bound for Z. (You can solve the LP's by graphically.)

6. Consider the following integer program:

$$\begin{aligned}
&\text{Max } Z = 3x + 2y \\
&\text{subject to} \\
&2x + 5y \leq 9 \\
&4x + 2y \leq 9 \\
&x, y \geq 0 \\
&x, y \text{ are integers}
\end{aligned}$$

Use Branch and Bound to solve the integer program. Show the branching tree that results when you perform the algorithm, and clearly label the vertices of the tree with the solution to the corresponding LP and the resulting bound for Z. (You can solve the LP's graphically.)

7. Consider the following nonlinear program:

$$\text{Maximize } f(x) = -2x^6 - 3x^4 + 10x^3 - 12x^2 + 60x$$

(a) Use the bisection method to (approximately) solve this problem. Use an error tolerance of $\epsilon = .07$ and use starting bounds $\underline{x} = 1$ and $\overline{x} = 2$.

(b) Use Newton's method to (approximately) solve this problem. Use an error tolerance of $\epsilon = 0.02$ and $x_1 = 1$.

8. Consider the following function:

$$f(x) = e^{-y}(x^2 - y^2)$$

(a) Find all critical points for this function.

(b) For each critical point, determine whether it is a local max, local min, or neither.

9. Consider the following nonlinear program:

$$\text{Maximize } f(x) = 6x_1 + 2x_1x_2 - 2x_2 - 2x_1^2 - x_2^2$$

(a) Show that f is concave.

(b) Perform two iterations of gradient search starting from $(x_1, x_2) = (0, 0)$ to find the approximate solution.

10. Use Lagrange multipliers to solve the following nonlinear program:

$$\begin{array}{l} \text{Max } x^2 + 2y^2 - 4x \\ \text{subject to} \\ x^2 + 4y^2 = 36 \end{array}$$

11. The Karush-Kuhn-Tucker conditions are:

   1. $\dfrac{\partial f}{\partial x_j} - \displaystyle\sum_{i=1}^{m} u_i \dfrac{\partial g_i}{\partial x_j} \leq 0$ for $j = 1, \ldots, n$
   2. $x_j^* \left(\dfrac{\partial f}{\partial x_j} - \displaystyle\sum_{i=1}^{m} u_i \dfrac{\partial g_i}{\partial x_j} \leq 0 \right) = 0$ for $j = 1, \ldots, n$
   3. $g_i(x^*) - b_i \leq 0$ for $i = 1, \ldots, m$
   4. $u_i \left(g_i(x^*) - b_i \right) = 0$ for $i = 1, \ldots, m$
   5. $x_j^* \geq 0$ for $j = 1, \ldots, n$
   6. $u_i \geq 0$ for $i = 1, \ldots, m$

   Use the Karush-Kuhn-Tucker conditions to solve the following nonlinear program:

$$\begin{array}{l} \text{Max } 2x_1 + 3x_2 - x_1^2 - x_2^2 \\ \text{subject to} \\ x_1 + x_2 \leq 2 \\ x_1, x_2 \geq 0 \end{array}$$