MATH 142: Practice Exam 3

Show all appropriate work. Variables may represent any real number.

1. Let $f(x,y) = \frac{1}{x^2+y^2+1}$.
   (a) Sketch the contour plot for f.
   (b) Find all second order partial derivatives of f.
2. Find the specified partial derivatives:
   (a) $f(x,y) = x^2y + \frac{x}{y}$, f_x and f_y.
   (b) $g(x,y) = x\cos(xy)$, g_x and g_y.
   (c) $h(x,y) = xye^{xy}$, h_{xy}.
3. Define $\int_c^\infty \int_a^b f(x,y)\,dxdy = \lim_{d\to\infty} \int_c^d \int_a^b f(x,y)\,dxdy$. Determine if $\int_1^\infty \int_1^2 \frac{1}{(x+y)^2}\,dxdy$ is convergent or divergent. If it converges, find its value. If it diverges, explain.
4. Evaluate the following integrals:
   (a) $\int_0^1 \int_0^1 (xe^y + x^2y - \cos y)\,dxdy$.
   (b) $\int_0^1 \int_0^1 xye^x\,dydx$.
   (c) $\int_0^1 \int_{-1}^1 \frac{x^2}{y^2} e^{\frac{x}{y}}\,dydx$.