MATH 142: Practice Midterm 1

Show all appropriate work. Variables may represent any real number.

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1. (a) Use integration by parts with $dv = \cos(x)\ dx$ to prove the reduction formula $\int cos^n(x)\ dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\ dx$.

   (b) Use your answer to part (a) to find $\int_0^{\frac{\pi}{2}} cos^5(x)\ dx$.

2. Let $f(x) = 2\cos\left[(2x - x^2)\frac{\pi}{2}\right]$ and $g(x) = 2x\cos\left[(2x - x^2)\frac{\pi}{2}\right]$. Given that $f(x) \geq g(x)$ on $[0, 1]$, find the area bound by $f(x)$ and $g(x)$ on the interval from 0 to 1.

3. Evaluate the following integrals:

   (a) $\int \frac{x+2}{(x^2+4x-3)^3}\ dx$.

   (b) $\int e^{-y}\cos y\ dy$.

   (c) $\int \sqrt{x}e^{\sqrt{x}}\ dx$.

   (d) $\int \frac{x^4+x^3-x^2-x+1}{x^3-x}\ dx$.