MATH 242: Practice Exam 1

Show all appropriate work. Variables may represent any real number.

1. (a) Suppose M is a 20×20 matrix that can be described as a block matrix

$$M = \begin{pmatrix} A & 0 \\ X & B \end{pmatrix},$$

where A and B are invertible matrices and 0 is a matrix of appropriate size with every entry zero. Write a formula for M^{-1} as a block matrix.

   (b) Write the 3×3 matrix E such that EA is the matrix obtained from A by subtracting row 1 from row 3, and leaving rows 1 and 2 unchanged.

2. Let

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 4 \\ 3 & 0 & 1 \end{pmatrix}.$$

   (a) Put A in upper triangular form

   (b) Factor A as $A = LU$.

   (c) What is the determinant of A?

3. $T : \mathbf{x} \mapsto \mathbf{y}$ means that the function takes the input $\mathbf{x}$ to the output $\mathbf{y}$. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T : \left(\begin{smallmatrix} 4 \\ 2 \end{smallmatrix}\right) \mapsto \left(\begin{smallmatrix} 2 \\ 6 \end{smallmatrix}\right)$ and $T : \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right) \mapsto \left(\begin{smallmatrix} 4 \\ 2 \end{smallmatrix}\right)$. Let A denote the standard matrix for T (this is the matrix such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$.).

   (a) The input-output data can be formulated as a matrix equation $AB = C$, whith $B = \left(\begin{smallmatrix} 4 & 1 \\ 2 & 1 \end{smallmatrix}\right)$. What is the matrix C?

   (b) Compute A.

4. Which of the following matrices are invertible? No justification required.

$$A = \begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 8 & -3 & 0 \\ 0 & 6 & 7 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & 0 & 8 \\ -1 & 7 & 6 & 0 \\ 2.1 & 0 & 4 & 4 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 4 & 1 & 3 & 8 \\ 7 & 1 & 0 & 6 & 0 \\ 0 & 5 & 2 & 1 & 1 \end{pmatrix}$$

5. For the following, answer TRUE or FALSE. No explanation required.

   (a) If A, B are matrices and column 1 of B is the zero column, then column 1 of AB is the zero column.

   (b) If $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$ are vector in $\mathbb{R}^3$, and none of them are scalar multiples of the others, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly independent.

   (c) If A, B, C are matrices such that $AB = AC$, then $B = C$.

   (d) Suppose A is an $m \times n$ matrix, $\mathbf{b}$ is a column vector in $\mathbb{R}^m$, and the vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ are solutions to the equation $A\mathbf{x} = \mathbf{b}$. Then there is a vector $\mathbf{w}$ that is a solution to $A\mathbf{x} = \mathbf{0}$ such that $\mathbf{v} = \mathbf{u} + \mathbf{w}$.