MATH 242: Practice Exam 2

Show all appropriate work. Variables may represent any real number.

1. The 4×6 matrix

$$R = \begin{pmatrix} 1 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 3 \\ 0 & 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

can be obtained by elementary row operations from the matrix

$$A = \begin{pmatrix} 5 & 10 & 3 & 5 & 17 & 9 \\ 2 & 4 & 1 & 3 & 4 & 7 \\ 3 & 6 & 2 & 4 & 9 & 10 \\ 2 & 4 & 1 & 2 & 6 & 3 \end{pmatrix}.$$

   (a) Find a basis for the column space of A.

   (b) Find a basis for the row space of A.

   (c) Find a basis for the Null space of A.

   (d) Which are the free variables?

2. Find the complete solution to

$$\begin{pmatrix} 1 & 3 & 2 & 4 \\ 2 & 6 & 6 & 6 \\ 0 & 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}.$$

3. Are the following subspaces? Justify your answer.

   (a) The set of all vectors in $\mathbb{R}^3$ perpendicular to $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

   (b) The set of all 2×2 matrices with determinant 0.

   (c) The line $y = 5$ in the plane.

4. (a) Are the following vectors independent or dependent?

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}.$$

   (b) Give an example of a matrix A, so that $A\mathbf{x} = \mathbf{b}$ will have 0 or 1 solution. Give an example of a $\mathbf{b}$ so that there is no solution and example for which there is a solution.