MATH 301: Written Assignment 1 **Due: Tuesday, September 18, 2018**

Show all appropriate work.

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1. Is the double precision floating point description of 0.2 larger, smaller or equal to 0.2?

2. How do you accurately evaluate $\sqrt{1+x} - \sqrt{1-x}$ when $x \ll 1$?

3. How do you accurately evaluate $\frac{1-\cos(x)}{\sin(x)}$ when $x \ll 1$?

4. (We will go over this recursion relation and its instability in Tuesday's class. It is also covered in the class.) Running the recurrence $E_n = 1 - nE_{n-1}$ forward is an unstable way to compute $E_n = \int x^n e^{x-1}\, dx$. However, we can get good results by running the recurrence backward from the estimate $E_n \approx \frac{1}{N+1}$ starting at large enough N. Explain why. How large must N be to compute E_{20} to near machine precision?

5. How would you accurately compute the function

$$f(x) = \sum_{j=0}^{\infty} (\cos(x^j) - 1),$$

for $|x| < 1$.

6. (To be done after Tuesday's class) Suppose we launch an object at an angle θ with an initial speed of v_0. Ignoring factors like air resistance and terrain, the object will land at distance

$$d = \frac{v_0^2}{g}\sin(2\theta),$$

where g is the acceleration due to gravity on earth (about 9.8 m/s). Suppose we compute the velocity needed to land a hundred meters away, and our launcher is pointed in exactly the right direction with a known launch angle.

   (a) What is the condition number for d as a function of v_0?

   (b) Suppose the launch angle has negligible error, but there is a 1% error in the launch velocity. If the target is a meter in radius, will we hit it?