MATH 301: Written Assignment 2 **Due: Thursday, September 27, 2018**

Show all appropriate work.

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1. Suppose A is an $m \times n$ matrix, $\mathbf{b}$ is a column vector in $\mathbb{R}^m$, and the vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ are distinct solutions to the equation $A\mathbf{x} = \mathbf{b}$. Show that there exists a nonzero solution to $A\mathbf{x} = \mathbf{0}$.

2. Suppose M is a 20×20 matrix that can be described as a block matrix
$$M = \begin{pmatrix} A & 0 \\ X & B \end{pmatrix},$$
where A and B are invertible matrices and 0 is a matrix of appropriate size with every entry zero. Write a formula for M^{-1} as a block matrix.

3. (a) Find the determinant of $A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 7 & 3 & 4 & 2 \\ 10 & 2 & 5 & 3 \end{pmatrix}$.
   (b) If B is a 3×3 matrix with $\det(B) = 2$, find $\det(3B)$, $\det(B^2)$ and $\det(-B)$.

4. For the following problems L is a lower triangular matrix, and U is an upper triangular matrix. If E is the product of the elimination matrices you applied to A to make it upper triangular then $EA = U$, and $L = E^{-1}$ is lower triangular.
   (a) Factor the matrix $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ as $A = LU$.
   (b) Let
   $$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 4 \\ 3 & 0 & 1 \end{pmatrix}.$$
      i. Put A in upper triangular form
      ii. Factor A as $A = LU$.
   (c) Let
   $$A = \begin{pmatrix} 2 & 1 & 2 \\ 4 & 1 & 3 \\ 2 & 2 & 1 \end{pmatrix}.$$
   Factor A as $A = LU$.

   If the following problems are not review, let me know.

5. Are the following vectors independent or dependent?
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}.$$

6. Find a basis for the subspace of $\mathbb{R}^5$ spanned by $\begin{pmatrix} 1 \\ 1 \\ -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -7 \\ 0 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \\ -3 \\ 6 \\ 9 \end{pmatrix}$.

7. Let V and W be subspaces of $\mathbb{R}^n$ such that $\text{dimension}(V) + \text{dimension}(W) > n$. Show that some nonzero vector is in both V and W.