MATH 352: Written Assignment 1 **Due: Friday, February 7, 2020**

Show all appropriate work.

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1. How long did the assignment take you?

2. Let $f(x, y, z) = xe^{2yz}$.

   (a) Find the gradient of f.

   (b) Evaluate the gradient at the point $P = (3, 0, 2)$.

   (c) Find the rate of change of f at P in the direction of $\mathbf{u} = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$. (Recall that the derivative of f in the direction of the unit vector $\mathbf{u}$ is $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$.)

3. Find the maximum rate of change $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 6, -2)$ and the direction in which it occurs.

4. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

5. Let u and v be differential functions of x and y. Show that $\nabla(uv) = u\nabla v + v\nabla u$.

6. The **second directional derivative** of $f(x, y)$ is

$$D^2_{\mathbf{u}}f = D_{\mathbf{u}}[D_{\mathbf{u}}f(x, y)].$$

   (a) If $\mathbf{u} = \langle a, b\rangle$ is a unit vector and f has continuous second partial derivatives, show that

$$D^2_{\mathbf{u}}f = \frac{\partial^2 f}{\partial x^2}a^2 + 2\frac{\partial^2 f}{\partial x \partial y}ab + \frac{\partial^2 f}{\partial y^2}b^2.$$

   (b) Find the second directional derivative of $f(x, y) = xe^{2y}$ in the direction of $\mathbf{v} = \langle 4, 6\rangle$.

7. If $f(x, y) = xy$, find the gradient vector $\nabla f(3, 2)$ and use it to find the tangent line to the level curve $f(x, y) = 6$ at the point $(3, 2)$.

8. (a) Two surfaces are called **orthogonal** at a point of intersection if their normal lines are perpendicular at that point. Show that surfaces with equations $F(x, y, z) = 0$ and $G(x, y, z) = 0$ are orthogonal at a point P where $\nabla F \neq \mathbf{0}$ and $\nabla G \neq \mathbf{0}$ if and only if

$$\frac{\partial F}{\partial x}\frac{\partial G}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial G}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial G}{\partial z} = 0 \text{ at } P.$$

   (b) Use (a) to show that the surfaces $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = r^2$ are orthogonal at every point of intersection. Can you see why this is true without using calculus?

9. Find the divergence and curl of $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + yze^x\mathbf{k}$.

10. Show that the any vector field of the form $\mathbf{F}(x, y, z) = f(x)\mathbf{i} + g(y)\mathbf{j} + h(z)\mathbf{k}$ where f, g, h are differentiable functions, is irrotational.

11. Prove the following identities, assuming the appropriate partial derivatives exist and are continuous:

(a) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

(b) $\nabla \cdot (f(x, y, z)\mathbf{F}(x, y, z)) = f(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla f$.

(c) $\nabla \cdot (\nabla f \times \nabla g) = 0$.