MATH 352: Written Assignment 5 **Due: Friday, March 13, 2020**

Show all appropriate work.

Reading

Read pages 31 through 40 on manifolds in Sean Carroll's lecture notes. If you have extra free time and are interested, feel free to continue reading through the paragraph that finishes after eqn. (2.27) on page 47 as that is what we will cover next, but be sure to at least read through page 40. Once we cover manifolds we will return to tangent spaces and make the concepts of vectors, tensors and the metric more precise.

Exersices

1. Consider a barn that is 10 m long and a pole that is 20 m long when measured in the same rest frame. The barn has a doors at the front and back. With the front door open and the back door closed Robin runs toward the barn holding the pole horizontally at $0.9c$. Robin's friend Jordan is standing outside the barn watching this happen. When Jordan measures the length of the pole which they see moving at $0.9c$, its length is contracted by a factor 2.29 so it will fit entirely in the barn. As soon as the back of the pole crosses the front door of the barn Jordan closes the front door so that for a moment the pole is completely contained in the closed barn. Jordan then immediately opens the back door allowing Robin to run through. From Robin's perspective, they and the pole are at rest and the barn is moving towards them with velocity $-0.9c$. Therefore Robin sees the length of the barn contract by a factor of 2.29 so that the pole will never fit in the barn. Explain what happens from Robin's perspective. Will the pole crash through the barn door? Include spacetime diagrams for both Jordan's and Robin's reference frame in your answer.

2. Imagine the space (not spacetime) is a finite box, or in more technical mathematical terms a 3-torus of size L. This means there is a coordinate system $x^\mu = (t, x, y, z)$ such that every point with coordinates (t, x, y, z) is identified with (equivalent to) every point with coordinates $(t, x + L, y, z)$, $(t, x, y + l, z)$ and $(t, x, y, z + L)$. Note that the time coordinate is the same. Now consider two observers; observer A is at rest in the coordinate system (has constant spatial coordinates), while observer B moves in the x-direction with constant velocity v. A and B start at the same event, and while A remains still, B moves once around the universe and comes back to intersect the worldline of A without ever having to accelerate (since the universe is periodic). What are the relative proper times experienced in the interval by A and B? Is this consistent with your understanding of Lorentz invariance?

3. Let R be a (k, l) tensor and T be an (m, n) tensor. Show that $R \otimes T$ is a multilinear map.