Show all appropriate work.

1. State the order of the following differential equations and whether it is linear or nonlinear:

   (a) $y' + e^x y = y^4$.

   (b) $y'' - 3yy' = x$.

   (c) $y^{(3)} = (\sin x)y^{(2)} + y = x$.

   (d) $x^2 y'' + y' + (\ln x)y = 0$.

2. Verify that $y(x) = x$ is a solution to the differential equation $y'' + y = x$.

3. Verify that $y(x) = x + C\sin x$, where C is a constant, is a solution to the differential equation $y'' + y = x$.

4. Which of the following functions satisfy the differential equation $y'' + 4y' + 5y = 0$:

   (a) e^{-2x}.

   (b) $e^{-2x}\sin(2x)$.

   (c) $e^{-2x}\cos(2x)$.

   (d) $\cos(2x)$.

5. Find the values of r so that $y(x) = xe^{rx}$ solves the differential equation $y'' + 4y' + 4y = 0$ on $(-\infty, \infty)$.