
Show all appropriate work.

1. Solve the following differential equations.
 - (a) $y' + y = e^x$.
 - (b) $y' - 2xy = e^{x^2}$.
 - (c) $y' + 2y = xe^{-2x}$ $y(1) = 0$.
 - (d) $\frac{dy}{dx} + \frac{y}{x} = \frac{\cos x}{x}$, $y(\frac{\pi}{2}) = \frac{4}{\pi}$, $x > 0$.
2. Consider the homogenous differential equation $\frac{dy}{dx} + P(x)y = 0$.
 - (a) Show that $y = 0$ is a solution (this is called the *trivial* solution).
 - (b) Show that if $y = y_1(x)$ is a solution and k is a constant, then $y = ky_1(x)$ is also a solution.
 - (c) Show that if $y = y_1(x)$ and $y = y_2(x)$ are both solutions, then so is $y = k_1y_1(x) + k_2y_2(x)$ is also a solution for k_1 and k_2 constants.
3. A pond that initially contains 500,000 gal of unpolluted water has an outlet that releases 10,000 gal of water per day. A stream flows into the pond at 12,000 gal/day containing water with a concentration of 2 g/gal of a pollutant. Find a differential equation that models this process, and determine what the concentration of pollutant will be after 10 days.