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Show all appropriate work.

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1. (a) Show that

$$n(1+x)^{n-1} + xn(n-1)(1+x)^{n-2} = \sum_{k=0}^n \binom{n}{k} k^2 x^{k-1}$$

by taking appropriate derivatives of  $(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k$  with respect to  $x$ .

- (b) Use your answer from part (a) to find a closed form formula for

$$\binom{n}{1} - 8\binom{n}{2} + 36\binom{n}{3} - 128\binom{n}{4} + \dots$$

*Note:* This is not an infinite sum.

2. How many “words” of length 5 can be constructed from the letters in the word BANANA.
3. Problems from the book:
- (a) Section 3.4: 8, 9.
- (b) Section 3.5: 3, 7.