

## ERRATA FOR

“Proofs and Fundamentals: A First Course in Abstract Mathematics”

Second edition

Ethan D. Bloch

Springer-Verlag, 2010

Last Updated February 22, 2024

Below is an updated list of errata. The fault for all the errors in the book is my own, and I offer my sincere apologies for any inconvenience caused by the errors in the book.

This list was compiled with the generous assistance of: Tilman Bauer, Taylor Boone, Joan Nicolas Castro Cortes, César Hernández Cruz, Daniel Cunningham, Eugene Dorokhin, Jenna Galka, Filipe Gomes, Beth Hoffman-Patalona, Ammar Khanfer, David Lou, Alex Lowe, Brendan Macmillan, David Makinson, Darren A. Narayan, George Vaughan, Rayo Verweij, Paul Weemaes, Japheth Wood.

If you find any additional errors in the book, or any errors in this list of errors, I would very much appreciate it if you would let me know by email at [bloch@bard.edu](mailto:bloch@bard.edu).

Page	Line/Item	Text	Comment/Should be
6	Line 2	“and $x < 2$ ”	Should be “and therefore $x < 2$ ”
8	Line -15	“ $P \wedge Q$ ”	Should be “ $P \rightarrow Q$ ”
16	Line -3	“is the same”	Should be “are the same”
20	Line -15	“the statement $[P \vee (Q \wedge R)] \leftrightarrow [(P \vee Q) \wedge (P \vee R)]$ ,”	Should be “the statement $[P \vee (Q \wedge R)] \leftrightarrow [(P \vee Q) \wedge (P \vee R)]$ is always true,”
21	Line -8	“note that, however, that”	Should be “note, however, that”
34	Line -19	“theorem”	Should be “theorems”
36	Line 7	“the $P(x)$ ”	Should be “ $P(x)$ ”
38	Line 18	“all types of ,”	Should be “all types of fruit,”
39	Line -10	“it it”	Should be “in it”
41	Line -16	“of $U$ ”	Should be “in $U$ ”
43	Exercise 1.5.4(3)	“is not riper”	Should be “are not riper”
44	Exercise 1.5.6(3)	“equation”	Should be “inequality”

49	Line 5	“Second”	Should be “Third”
52	Line -2	“proofs”	Should be “proof”
63	Paragraph 3		Replace the entire paragraph with “Though we have proved that there are infinitely many prime numbers, our proof did not produce an explicit infinite list of prime numbers, but only proved that in theory such a list exists. More generally, we distinguish between a constructive proof, which proves the existence of something by actually producing it, and an existence proof (such as the above proof), which only shows that in theory something exists. Existence proofs are one of the hallmarks of modern mathematics. Though most mathematicians today accept existence proofs, there is a minority who do not accept their validity. See [Ang94, Chapter 39], [GG94, Section 5.6] and [EC89, Chapter 26] for a discussion of the mathematical philosophies known as “intuitionism” and “constructivism,” which differ from mainstream mathematics.”
82	Line 13	“the student”	Should be “he”
82	Lines 29–23	“A proof is an explanation of why something is true. A well-written proof is an explanation that someone else can understand.”	Should be “A proof is an argument showing that something is true. A well-written proof is an argument that someone else can understand.”
86	Line -3	“ $xy < 0$ ”	Should be “ $xy \leq 0$ ”
86	Line -1	“ $xy < 0$ ”	Should be “ $xy \leq 0$ ”
87	Line 8	“only then”	Should be “then only”
93	Line -6	“young age).”	Should be “young age.)”
95	Line 9	“ $A \not\subset B$ ”	Should be “ $A \not\subseteq B$ ”

99	Exercise 3.2.5(3)	“The set of all rational numbers that have a factor of 5 in their denominators.”	Should be “The set of all rational numbers that have denominators that are divisible by 5.”
100	Exercise 3.2.13	“ $\mathcal{P}(A)$ ?”	Should be “ $\mathcal{P}(A)$ .”
108	Exercise 3.3.10	“ $C \subset A \cup B$ ”	Should be “ $C \subseteq A \cup B$ ”
111	Line 5	“some $i \in \mathbb{N}$ ”	Should be “all $i \in \mathbb{N}$ ”
111	Definition 3.4.2		<p>Replace the entire definition with “<b>Definition 3.4.2</b></p> <p>Let <math>\mathcal{A}</math> be a set.</p> <ol style="list-style-type: none"> <li>1. The set <math>\mathcal{A}</math> is called a <b>family of sets</b> if all the elements of <math>\mathcal{A}</math> are sets.</li> <li>2. Suppose that <math>\mathcal{A}</math> is a family of sets. Let <math>I</math> be a set. The family of sets <math>\mathcal{A}</math> is <b>indexed</b> by <math>I</math>, denoted <math>\mathcal{A} = \{A_i\}_{i \in I}</math>, if for each <math>i \in I</math> there is a unique element of <math>\mathcal{A}</math> denoted <math>A_i</math>, and if every element of <math>\mathcal{A}</math> equals <math>A_k</math> for some <math>k \in I</math>.”</li> </ol>
112	After the first paragraph		<p>Add the paragraph “A close look at the definition of a family of sets reveals that there is one important difference between families of sets that are non-indexed and those that are. A non-indexed family of sets <math>\mathcal{A}</math> is itself a set, and elements in a set are listed only once each. Hence, a set that is an element of <math>\mathcal{A}</math> can be listed only once in <math>\mathcal{A}</math>. By contrast, suppose that <math>\mathcal{A}</math> is indexed by a set <math>I</math>, so that <math>\mathcal{A} = \{A_i\}_{i \in I}</math>. According to the definition of an indexed family of sets, it is possible that <math>A_i = A_j</math> for some <math>i, j \in I</math> such that <math>i \neq j</math>. Such repetition is sometimes convenient because it allows us to make use of an indexing for a family of sets that arises in some natural way.”</p>
113	Line -3	“ $x \in \bigcup_{i \in I} A_i$ ”	Should be “ $y \in \bigcup_{i \in I} A_i$ ”
115	Line 2	“ $\bigcap_{X \in \mathcal{A}} X \subseteq \bigcap_{Y \in \mathcal{B}} Y$ ”	Should be “ $\bigcap_{X \in \mathcal{A}} X \supseteq \bigcap_{Y \in \mathcal{B}} Y$ ”
116	Line -2	“such such”	Should be “such”
123	Line -10	“The family $\mathcal{C}$ is a <b>chain</b> ”	Should be “The family $\mathcal{C}$ is a <b>chain</b> in $\mathcal{P}$ ”

124	Theorem 3.5.6 (Zorn's Lemma)		There is an error in the proof of Zorn's Lemma. A valid proof may be found at <a href="http://faculty.bard.edu/bloch/zorns_lemma_proof.pdf">http://faculty.bard.edu/bloch/zorns_lemma_proof.pdf</a>
127	Line 3	"Principle"	Should be "Theorem"
144	Exercise 4.2.4	"for all $(x,y) \in \mathbb{R}$ "	Should be "for all $(x,y) \in \mathbb{R}^2$ "
151	Line -12	"by from"	Should be "by"
151	Line -4	"for all $x \in \mathbb{R}$ "	Should be "for all $x \in (0, \infty)$ "
152	Exercise 4.3.5	"Let $A$ and $B$ be sets"	Should be "Let $A, B$ and $C$ be sets"
158	Line -13	" $(g \circ f)(a) = p$ "	Should be " $(g \circ f)(a) = c$ "
172	Line 7	"we write $a R b$ if $(a,b) \in R$ "	Should be "we write $a R b$ if $(a,b) \in \bar{R}$ "
187	Line 18	" $a \sim y$ "	Should be " $x \sim y$ "
193	Exercise 5.3.12		In the commutative diagram change " $f$ " to " $h$ "
198	Commutative diagram		In the commutative diagram change " $H$ " to " $A$ " in two places
199	Line -13	"Parts (12) and (15)"	Should be "Parts (10), (12) and (15)"
218	Line 10	"We will shown"	Should be "We will show"
228	Line 17	" <b>Proof Theorem 6.5.10</b> "	Should be " <b>Proof of Theorem 6.5.10</b> "
228	Line -4	"to proved"	Should be "to provide"
229	Line -21	" $CC$ "	Should be " $C$ "
229	Line -17	" $\bigcup_{F \in C} C$ "	Should be " $\bigcup_{F \in C} F$ "
232	Line -15	"this section"	Should be "Section 6.5"

243	Line 28–30	“By Theorem 6.7.1 and Theorem 6.6.9 (2) we see that $\mathbb{Q} \cup P$ is countable, and by Lemma 6.5.5 (2) and Corollary 6.6.6 we see that $\mathbb{Q} \cup P$ is countably infinite.”	Should be “By Theorem 6.7.1 and Lemma 6.5.5 (2) we know that $\mathbb{Q}$ is both countable and infinite. Hence, Theorem 6.6.9 (2) implies that $\mathbb{Q} \cup P$ is countable. Because $\mathbb{Q} \cup P$ contains an infinite set, it follows from Corollary 6.6.6 that $\mathbb{Q} \cup P$ is infinite. Because $\mathbb{Q} \cup P$ is both countable and infinite, then it is countably infinite.”
247	Line 21	“rational rational”	Should be “rational”
251	Line -5	“the properties”	Should be “their properties”
259	Line -4	“it will follow that $a' * b'$ ”	Should be “it will follow that $b' * a'$ ”
261	Line 20	“ $H \subset G$ ”	Should be “ $H \subseteq G$ ”
280	Exercise 7.4.13	“let $h: X \rightarrow A$ be a function”	Should be “let $h: X \rightarrow A$ be an injective function”
317	Line -7	“case case”	Should be “case”
321	Line 10	“that that”	Should be “that”
342	Line -15	“real number”	Should be “real numbers”