ERRATA FOR

"The Real Numbers and Real Analysis"
Ethan D. Bloch
Springer-Verlag, 2011

Last Updated: September 23, 2022

Below is an updated list of errata. The fault for all the errors in the book is my own, and I offer my sincere apologies for any inconvenience caused by the errors in the book.

This list was compiled with the generous assistance of: Jerónimo Alaminos, Eduardo Bravo, Adam Dingle, Ahmad Khaled, Greg Landweber, Wai Wah Lau, Vho Lhanjiao, Einam Livnat, Richard Mitchell-Lowe, Chris Murphy, Michael Naumov, Daniel Reyes, Yahia Saleh, Luis Serra, Vishrut Tiwari, Kristaps Treigūts, Guido Ursoleo, Umair Ali Wani, Yuanhong Zhi.

If you find any additional errors in the book, or any errors in this list of errors, I would very much appreciate it if you would let me know by email at `bloch@bard.edu`.

Page	**Line/Item**	**Text**	**Comment/Should be**
xiii	Line -7	"it it"	Should be "it is"
8	Line 13	"Let $a,b,c,d \in \mathbb{N}$"	Should be "Let $a,b,c \in \mathbb{N}$"
13	Lines 11, 14, 15	"$[(b,d)]$"	Should be "$[(c,d)]$"
13	Lines 13, 14, 15	"$[(y,w)]$"	Should be "$[(z,w)]$"
14	Line -2	"$p \neq 1$"	Should be "$p+1 \neq 1$"
14	Line -1	"$[(p+1),1] > \hat{0}$"	Should be "$[(p+1,1)] > \hat{0}$"
16	Line -7	"**Lemma 1.3.8**"	Should be "**Theorem 1.3.8**"
16	Lines (-9)–(-7)	"Whereas the proof of Theorem 1.3.8 makes use of only the properties of the integers given in Theorem 1.3.5, it turns out that not all properties of the integers can be deduced from that theorem."	Should be "Whereas the proof given above of various parts of Theorem 1.3.8 makes use of only the properties of the integers given in Theorem 1.3.5, it turns out that not all properties of the integers can be deduced from that theorem, for example Theorem 1.3.8 (7), the proof of which makes use of Theorem 1.2.7 (13)."
22	Line -15	"From the definition of $\mathbb{N}$, we observe that $S \subseteq \mathbb{N}$."	Should be "From the definition of $\mathbb{Z}$, we observe that $S \subseteq \mathbb{Z}$."

25	Lemma 1.4.5 (7)		The proof of Part (7) of this lemma ultimately requires the Well-Ordering Principle (because this part of the lemma is not satisfied by the rational numbers), and so it cannot be proved at this point, but rather should be proved using Exercise 1.4.5 (which is proved using Theorem 1.4.6, which in turn is proved using the Well-Ordering Principle), together with various other parts of Lemma 1.4.5.
25	Exercise 1.4.7	"The relation $<$, called the **dictionary order** on $\mathbb{Z}[x]$, is defined by $f < g$ if and only if either the degree of f is less than the degree of g, or if the degrees of f and g are equal and if $f \neq g$ and if the highest degree coefficient which differs for f and g is smaller for f, for all $f, g \in \mathbb{Z}[x]$."	Should be "The relation $<$, called the **dictionary order** on $\mathbb{Z}[x]$, is defined by $f < g$ if and only if for the highest degree coefficient on which f and g differ, the coefficient is smaller for f, for all $f, g \in \mathbb{Z}[x]$."
25	Exercise 1.4.7 (2)	"$f < g < g+1$"	Should be "$f < g < f+1$"
28	Line 12	"or $xw > yz$"	Should be "and $xw > yz$"
33	Line 7	"$r^2 < p$"	Should be "$r^2 < s$"
33	Line 8	"$\left(r+\frac{1}{k}\right)^2 < p$"	Should be "$\left(r+\frac{1}{k}\right)^2 < s$"
37	Line 18	"Hence $y \in B$."	Should be "Hence $z \in B$."
45	Lines 2–5	"There is some $p \in A - B$. Then by Lemma 1.6.5 (1) we know that $p < b$ for all $b \in B$. Let $c \in C$. Then $p + c < b + c$ for all $b \in B$. It follows from Lemma 1.6.5 (1) that $p + c \in \mathbb{Q} - (B + C)$. Because $p + c \in A + C$, we deduce that $A + C \supsetneqq B + C$."	Should be "Suppose that $A + C = B + C$. Then $(A + C) + (-C) = (B + C) + (-C)$. By Part (1) of this theorem we see that $A + (C + (-C)) = B + (C + (-C))$, by Part (4) it follows that $A + D_0 = B + D_0$, and by Part (3) we deduce that $A = B$, which is a contradiction. Hence $A + C \supsetneqq B + C$."
53	Line -3	"invent invent"	Should be "invent"

55	Line -5	"used proof induction in his discussion of what we call Pascal's triangle in 1665, and gave an explanation of this method of proof."	Should be "used proof by induction, and gave an explanation of this method, in his discussion of what we call Pascal's triangle around 1654 (published posthumously in 1665)."
71	Lemma 2.3.9 (7)	"$\|a\| - \|b\| \leq \|a+b\|$ and $\|a\| - \|b\| \leq \|a-b\|$"	Should be "$\|\|a\| - \|b\|\| \leq \|a+b\|$ and $\|\|a\| - \|b\|\| \leq \|a-b\|$"
74	Exercise 2.3.5	"[Used in Exercise 2.3.6 and Exercise 2.8.9.]"	Should be "[Used in Exercise 2.3.6.]"
80	Line -11	"**irrrational**"	Should be "**irrational**"
89	Line 2	"$x \in \mathbb{N}$"	Should be "$n \in \mathbb{N}$"
91	Line 4	"define define"	Should be "define"
97	Line -2	"X"	Should be "A"
97	Line -1	"X"	Should be "A"
100	Line 17	"dedcue"	Should be "deduce"
102	Line 6	"will follows"	Should be "will follow"
127	Exercise 2.8.2(2)	"Prove that $0 \leq q < y$."	Should be "Prove that $0 \leq q \leq y$."
128	Exercise 2.8.9	"[Use Exercise 2.3.5 (2) and Exercise 2.8.8.]"	Should be "[Use Exercise 2.8.8.]"
159	Line -4	"there is there is"	Should be "there is"
166	Line 15	"Property Property"	Should be "Property"
167	Line 5	"$L = F - Q$"	Should be "$L = F - U$"
171	Line 19	"f has the form $f(x) = a_0 + a_1x + \cdots + a_nx^n$"	Should be "p has the form $p(x) = a_0 + a_1x + \cdots + a_nx^n$"
171	Line 24	"$f(r) = 0$"	Should be "$p(r) = 0$"
177	Line -5	"discontinous"	Should be "discontinuous"
182	Lines (-6)–(-5)	"Suppose that $f\colon I \to \mathbb{R}$ is a function, for some open interval $I \subseteq \mathbb{R}$ be an open interval"	Should be "Let $f\colon I \to \mathbb{R}$ be a function, for some open interval $I \subseteq \mathbb{R}$"

189	Line 5	"exists all $i \in \mathbb{N}$"	Should be "exists for all $i \in \mathbb{N}$"
197	Line -3	"$f'(c) \neq 0$"	Should be "$f(c) \neq 0$"
198	Line 17	"Suppose that $b-a=c-b$"	Should be "Suppose that f is differentiable, that $b-a=c-b$"
204	Line 14	"antiderivative unique"	Should be "antiderivative is unique"
215	Exercise 4.5.5	"Suppose that f is differentiable at c."	Should be "Suppose that f is continuously differentiable on I."
236	Line -17	"practive"	Should be "practice"
240	Line -4	"$M \in \mathbb{N}$"	Should be "$M \in \mathbb{R}$"
251	Line 6	"$\lim_{w \to d^-} \int_c^{g^{-1}(g(w))} f(x)\,dx$"	Should be "$\lim_{w \to d^-} \int_c^{g(g^{-1}(w))} f(x)\,dx$"
255	Lines 4–5	"Suppose that f is strictly increasing."	Should be "Suppose that f is continuous and strictly increasing."
255	Line 9	Remove "Suppose that f is continuous at b."	
257	Line -8	"$P \in \mathbb{R}$"	Should be "$K \in \mathbb{R}$"
257	Line -8	"$\lvert f(x)-f(y)\rvert \leq P$"	Should be "$\lvert f(x)-f(y)\rvert \leq K$"
257	Line -7	"$M_i(f)-m_i(f) \leq P$"	Should be "$M_i(f)-m_i(f) \leq K$"
261	Line 19	"$[M,P]$"	Should be "$[P,M]$"
261	Line 20, two places	"$[M,P]$"	Should be "$[P,M]$"
272	Line 14	"$S=\{s_1,s_2,\ldots,s_n\}$"	Should be "$V=\{s_1,s_2,\ldots,s_n\}$"
272	Line 15	"S is a representative set"	Should be "V is a representative set"
272	Line 16	"$S(f,R,S)$"	Should be "$S(f,R,V)$"
272	Line 18	"S is a representative set"	Should be "V is a representative set"
272	Line 18	"$S(f,R,S)$"	Should be "$S(f,R,V)$"
274	Line -11	"continous"	Should be "continuous"
312	Line 2	"$\sqrt{[w_1-v_1]^2+[w_2-u_2]^2}$"	Should be "$\sqrt{[w_1-v_1]^2+[w_2-v_2]^2}$"

316	Line 9	“explictly”	Should be “explicitly”
318	Line -10	“sums sums”	Should be “sums”
330	Exercise 6.2.7	“Suppose that for each $M \in \mathbb{R}$, there is some $x \in I - \{c\}$ such that $f(x) > M$.”	Should be “Suppose that for each $M \in \mathbb{R}$ and each $\delta > 0$, there is some $x \in I - \{c\}$ such that $\lvert x - c \rvert < \delta$ and $f(x) > M$.”
342	Line 11	“define define”	Should be “define”
392	Line -8	“fof”	Should be “for”
410	Line 8	“$n \in \mathbb{R}$”	Should be “$n \in \mathbb{N}$”
410	Line -2	“convergnet”	Should be “convergent”
534	Line -7	“compendius”	Should be “compendious”